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Supply function Nash equilibrium of joint day-ahead electricity markets and forward contracts

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ABSTRACT

Forward contracts are one of the prevalent and useful tools for managing the risks associated with the volatility of the electricity market prices. Forward contracts and day-ahead electricity market are executed simultaneously, and hence, they affect each other. This paper proposes a comprehensive supply function equilibrium model to consider the mutual interactions between forward contracts and the associated day-ahead electricity market in a power system. Negotiation between each producer and consumer in forward market is taken into account in the model. In order to consider the risk management behaviors of all market players in the model, a new risk management method is presented. The proposed risk management method takes into account the concerns of market players about the future prices of day-ahead market over the delivery period. The model proposed in this paper is applied to a test system with forward and day-ahead markets. The results are compared with the case that there is no forward contract in the power system. Impacts of growing the concerns of producers and consumers about the future, impacts of increasing demand uncertainty, impacts of improving bargaining power of consumers in contracting period and impact of contracting obligations for producers on the simulation results are discussed. The proposed risk management method is compared with CVaR method and its efficiency is evaluated. Finally, applicability of the proposed model to real size power systems is examined.

1. Introduction

Restructuring of power industry has changed the manner of interaction between producers and costumers since the late twentieth century. The main goals of restructuring are increasing the investment of the private sector, creating a competitive and efficient market, promote technical growth and improve customer satisfaction [1]. In restructured environment, producers and consumers are free to choose between Pool-co wholesale electricity market and different types of long-term and short-term contracts to sell their produced power or buy their necessary power. Consumers are usually distribution companies, large industrial loads or retailers. In wholesale electricity market, producers and consumers submits their power and price bids to ISO and ISO determines MCP and scheduled power of each market player. Volatility of MCP is usually very high and this provides serious planning problems for both producers and consumers. In order to avoid the risks associated with MCP volatility, market players are willing to use financial tools, such as Forward contracts, options and futures [2,3].

Todays, a considerable volume of power in the power industry is traded through Forward contracts. Signing a forward contract between a producer and a consumers implies the producer to sell a given power quantity to the consumer throughout a pre-specified time period at a fixed price in $/MWh [4]. Day-ahead electricity markets are executed parallel with Forward contracts. On one hand, forecasted day-ahead MCP can be chosen as a guide for determining the contract price and on the other hand signing Forward contracts by different market players changes their behavior in the electricity market and affects the market prices [5]. Hence, they can have mutual impacts on each other. It's very important for market players and ISO to study these mutual impacts in order to increase their profit and electricity market efficiency, respectively.

The problem of mutual interaction between forward contracts and electricity markets has been studied before in the literature frequently. These studies can be categorized into two main groups: (1) market

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Abbreviations: MCP, Market Clearing price; ISO, Independent System Operator; MILP, Mixed Integer Linear Problem; α-CVaR, Conditional Value-at-Risk at α confidence level; SFE, Supply Function Equilibrium; CfD, Contract for Difference; PDF, Probability Density Function; MPEC, Mathematical Program with Equilibrium Constraint; EPEC, Equilibrium Problem with Equilibrium Constraint

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Nomenclature

A. Indices

i or u	power system producers
j or l	power system consumers
s	concern scenarios
t	hours of delivery period
d	days of delivery period

B. Sets

F	set of producers
J	set of consumers
S	set of concern scenarios
T	set of hours of delivery period
D	set of days of delivery period

C. Constants

a_i	intercept of marginal cost function of producer i
b_i^e/b_i^c	slope of marginal cost/bid function of producer i in day-ahead/forward market
$c_{j,s}^{td}$	intercept of marginal utility function of consumer j in delivery hour t of day d at concern scenario s
d_j^e/d_j^c	slope of marginal utility/offer function of consumer j in day-ahead/forward market
$\bar{Q}_i/\underline{Q}_i$	maximum/minimum output power of the producer i
Q_i^{up}/Q_i^{down}	ramp-up/ramp-down rate limits of producer i in per MW per hour
$\rho_{i,s}^{\pi,td}/\rho_{j,s}^{u,td}$	probability of concern scenario s at hour t of day d form the viewpoint of producer i/consumer j
M_{ij}^c/N_{ji}^c	very small/big positive number
τ/β	beta PDF shape parameters
v/w	lower/upper bound for Beta PDF
γ	weighting factor of the CVaR
θ	confidence level used in the calculation of the CVaR

D. Variables

α_{ij}^c	intercept of bid function of producer i to consumer j in forward contract market
ϵ_{ji}^c	intercept of offer function of consumer j to producer i in forward contract market
$\alpha_{i,s}^{e,td}$	intercept of bid function of producer i at concern scenario s and hour t of day d in day-ahead market
Q_{ij}^c/Q_{ji}^c	contracted power of producer i with consumer j/consumer j with producer i
Q_i^c/Q_j^c	total quantity of contracted power of producer i/consumer j
$Q_{i,s}^{e,td}/Q_{j,s}^{e,td}$	scheduled power of producer i/consumer j at concern scenario s and hour t of day d in day-ahead electricity market
$Q_{i,s}^{td}/Q_{j,s}^{td}$	sum of total quantity of contracted power of producer i/consumer j and scheduled power of producer i/consumer j at concern scenario s and hour t of day d in day-ahead electricity market
F_{ij}^c/F_{ji}^c	price of forward contract between producer i and consumer j/consumer j and producer i
λ_s^{td}	MCP at concern scenario s and hour t of day d in day-ahead electricity market
$\bar{\mu}_i^{ts}/\underline{\mu}_i^{td}$	lagrangian multiplier associated with maximum/minimum generation capacity of producer i at concern scenario s and hour t of day d
$\mu_{i,s}^{e,td}/\mu_{j,s}^{e,td}$	lagrangian multiplier associated with positivity of output power of producer i/consumer j at concern scenario s and hour t of day d in day-ahead market
μ_{ji}^c/μ_{ji}^c	lagrangian multiplier associated with minimum quantity of power for forward contract between producer i and consumer j/consumer j and producer i
$\mu_{i,s}^{up,td}/\mu_{i,s}^{do,td}$	lagrangian multipliers associated with ramp rates of output power of of producer i at concern scenario s and hour t of day d
ζ	auxiliary variable used to calculate the CVaR
η_s	auxiliary variable related to scenario s and used to calculate the CVaR

player viewpoint studies and (2) market regulator viewpoint studies. Refs. [6–11] study the problem from the viewpoint of the market player. Ref. [6] solves two separate optimizations for a consumer and a generation company for optimal power allocation between forward contracts with predefined prices and day-ahead market. Ref. [7] proposes a model for consuming a load through weekly and monthly forward contracts, buying from day-ahead electricity market and limited amount of self-production. Dynamic programming is used to solve the problem in [6] and [7]. Ref. [8] proposes a model for maximizing the profit of a single wind hydro-pump storage unit. This unit is able to trade energy with day-ahead electricity market and sell its power in predefined prices through forward contracts. A balancing market is also considered for charging the unit for deviating from scheduled power in day-ahead market. In Ref. [9] a stochastic model for decision making of a distribution company for the level of involvement in the forward contracts and day-ahead market is stablished. The proposed model considers demand response and load cut strategies, distribution generation by the units in distribution network and the competition between rival distribution companies for demand electricity pricing. Ref. [10] proposes a methodology that allows a producer to select weekly forward contracts, and obtain the offering strategy for day-ahead electricity market as a price-taker firm and derive the self-scheduling of its production units. In Ref. [11] a multi-stage stochastic model for selling strategy of a risk-averse producer in forward and option contracts and day-ahead electricity market is proposed. The common features of almost all market player viewpoint studies are as follows: (1) problem is modeled as a MILP model [6–11], (2) α-CVaR index is used to measure the risk [6–11], (3) forward contract prices are assumed known and fixed [6–11] and (4) since the forward contracts are signed for next few months, electricity market prices in power delivery period are uncertain and estimated using some forecast scenarios [6–11]. The most important points about these studies are that the mutual impacts of forward contracts and the electricity market prices in oligopoly structures are not considered and forward contract prices are assumed to be known and predefined.

Refs. [12–16] solve the problem from the viewpoint of the market regulator. Ref. [12] proposes an optimal operation model for an electricity market with simultaneous interaction of forward contracts, day-ahead market and reserve markets. It's assumed that the quantity and price of forward contracts are known and predefined, market players bid their marginal costs and necessary reserve power is certain, known and fixed. In Ref. [13] Nash equilibrium of a bilateral electricity market is examined. Day-ahead electricity market is ignored and contract parties are assumed to bid their marginal costs. Ref. [14] proposes a mathematical model for negotiation process of a forward contract between a load and a generator alongside with the day-ahead electricity market. Different definitions of risk and profit are considered in the model and an iterative algorithm is used to find the final negotiation results. SFE of an electricity market with forward bilateral contracts is investigated in Ref. [15]. Strategic behavior of the loads in the contract negotiation process, generation capacity of the units and effects of electricity market prices on the quantity of forward contracts are not considered in this model. Ref. [16] introduces a day-ahead and forward market including fuel-bases generators in a transmission network, suppliers as intermediaries and consumers with flexible and inflexible loads and renewable resources in a distribution network. It's assumed that the quantity of forward contracts are known and it is just necessary to find the contract prices. Objective functions are defined for each type of market players and an iterative algorithm is used to model the price adjustment process. Concentrating on these studies shows that none of these works present a proper and efficient structure for the problem. For instance, in Refs. [12] and [16] the role of consumers on determining the contract prices is ignored, in [16] decision variables are simplified and only one of quantity or price of the contract are considered as variables, in [14] other market players and their strategies are not considered in the model, and in [12] perfect competition model is assumed for day-ahead electricity market. Plus, none of these studies describe the mutual interaction between forward contracts and day-ahead market properly.

In this paper, a SFE model is presented for studying the impacts of parallel exercise of a forward contract market and the associated day-ahead electricity market on the forward contracts and day-ahead market prices and quantity of traded power in each market. The problem is solved from the viewpoint of market regulator. The main contributions of this paper are as follows:

(1) Proposing a comprehensive supply function equilibrium model to consider the mutual interactions between forward contracts and the associated day-ahead electricity market in a power system. Negation between each producer and consumer in forward market is taken into account in the model. Although the available research work have proposed models for considering negations among market players, they have ignored the impacts of these negotiations on the day-ahead market prices and vise-versa.

(2) Presenting a risk management method in order to consider the risk management behaviors of all market players in the proposed model. The presented risk management method is able to take into account the concerns of each producer and consumer about the increasing or decreasing of the market price, whereas the existing methods are not able to consider it. The presented method is compared with most common risk management method, i.e. CVaR, and efficiency of the presented risk management method is evaluated.

(3) Taking into account the assumptions that are not considered simultaneously in the literature. Considering these assumptions makes the problem more realistic. These assumptions are: a) considering strategic behavior of both producers and consumers in selecting forward contract partners, b) considering both price and volume of forward contracts as decision variables, c) assuming imperfect competition in electricity market that is more realistic for power systems, and d) considering all producers and consumers in problem solution (not just two contracted parties). In this condition, results show that considering forward market reduces the day-ahead market price and leads to contract prices which are lower than expected market price which benefits the consumers.

The paper is organized as follows. Problem definition and assumptions are presented in Section 2. In Section 3, a SFE model for joint forward and day-ahead electricity markets is presented. The proposed model is applied to a case study and the results are discussed in Section 4. Finally, Section 5 concludes the paper.

2. Problem definition and assumptions

2.1. Problem definition

Consider an oligopoly power system that consists of some fuel-based producers and consumers and suppose an imperfect competition among them. It is assumed that the market regulator allows the producers and consumers to take part in forward contracts through a forward contract market parallel with participating in day-ahead electricity market. In this situation, each producer or consumer tries to find an optimal power allocation in forward contracts and day-ahead electricity market that maximizes its aggregated profit.

In this paper, mutual impacts of forward contracts and day-ahead markets on day-ahead and forward contract prices and quantities are studied. To this end, optimal behavior of producers and consumers in contract negotiation process and day-ahead market should be determined. For this purpose, it is assumed that the forward contracts and day-ahead markets have reached to their Nash equilibrium. Hence, the main goal of this paper is to model the optimal behavior of producers and consumers in Nash equilibrium of a joint forward and day-ahead markets.

2.2. Sequence of actions

Timeline is divided into two separate periods, contracting period and delivery period. In contracting period both producers and consumers should decide about the quantity and price of their forward contracts for delivery period considering different future scenarios for total demand of each consumer and MCP in delivery period. In general, each producer/consumer can have a forward contract with every other consumers/producers. As soon as the negotiations are finalized and price and quantity of each contract are determined, forward contracts are concluded, and price and quantity of each contract are fixed and cannot be changed. Delivery period is supposed to be several hours of one or several consecutive days in future. Demand is different at different hours and different days of the delivery period. In delivery period, forward contracts should be settled by physical delivery and producers can sell the rest of their available energy in day-ahead market and consumers can buy the rest of their required energy from day-ahead electricity market. It should be noted, while in day-ahead market, price and power of a market player are different at each hour of each day of delivery period, price and power quantity of each contract are constant over all hours and days of its delivery period.

2.3. Concern scenario concept

Based on the Black Scholes pricing model [17], time interval between contracting period and delivery period, affects the uncertainty range and can be modeled by uncertainty range. The longer contract period means the wider range of uncertainty. Hence, a specific contract period is considered in this paper and the effect of contract period duration is modeled by the range of uncertainty. Different uncertain parameters such as total demand and fuel price affect forward and day-ahead markets prices and power quantities. It is assumed that fuel is purchased through long-term contracts and it is a deterministic variable in the delivery period. Precise forecast of demand of each consumer is not possible. Hence, different discrete *forecast scenarios* with specific probabilities are defined for demand of each consumer.

In this paper, a new risk management method is introduced. The proposed method is inspired by different concerns of producers and consumers. In fact, the main incentive of producers/consumers to participate in forward contracts is to relieve the risk of decreasing/increasing day-ahead market price and losing money. While the reality that will happen in future is unique, the viewpoints of the producers and consumers about future are not the same. They are not optimist about the variations of market price in future. Producers/consumers are worried about decreasing/increasing of demand in future and consequently decreasing/increasing of electricity market price. Hence, producers/consumers put more attention on the scenarios that lead to decreasing/increasing of market price. So, in this method, each producer/consumer assigns greater probabilities to the scenarios that lead to decrease/increase of market price. Assigning the greater probability to a scenario by a market player means the higher concerns of that market player for happening that scenario. These scenarios with the assigned probabilities by a market player considering his or her concerns are referred to as *concern scenarios* of that market player. Concern scenarios and their probabilities are generated as describes in the next subsections.

2.3.1. Concern scenarios generation

Assume that the marginal cost function of producer i and marginal utility function of the consumer j at hour t on day d of delivery period are $a_i + b_i^e Q_i^{td}$ and $c_j^{td} - d_j^e Q_j^{td}$, respectively. Demand variation in different hours is modeled by changing the intercept of utility functions of each consumer with Q_j^{td} axis i.e. c_j^{td}/d_j^e in different hours and days. In fact, increasing the demand of consumers is modeled by increasing the intercept of their utility functions. For the sake of simplicity, it is assumed that parameter d_j^e is constant and c_j^{td} is changed at each hour of each day. Consumers' demand at each hour is uncertain. Forecast scenarios for each consumer are generated by creating $n_s$discrete scenarios around the intercepts of marginal utility functions with Q_j^{td} axis at each hour. Again, it is assumed that parameter d_j^e is constant and c_j^{td} is changed at each scenario [18]. So, c_j^{td}, Q_i^{td}andQ_j^{td} turn into $c_{j,s}^{td}$, $Q_{i,s}^{td}$ and $Q_{j,s}^{td}$ for forecast scenario s in which $c_{j,s+1}^{td} \geq c_{j,s}^{td}$. It is assumed that uncertainty of demand of all consumers are affected by similar factors like economic or social issues. Hence, it is assumed that the correlation between demands of consumers is equal to 1. Forecasted demand scenario s for hour t of day d is introduced by set $c_s^{td} = \{c_{j,s}^{td}\ \forall\ j \in J\}$ where $c_{s+1}^{td} \geq c_s^{td}$. Other scenario generation methods can also be used to create these scenarios but all of them should satisfy the fact that $c_{j,s+1}^{td} \geq c_{j,s}^{td}$. Then, each market player assigns a probability to each forecast scenario considering his or her concerns and composes his or her concern scenarios.

2.3.2. Assignment of probabilities to concern scenarios

In the next step, the probabilities of total demand scenarios should be determined considering the different concerns of each producer and consumer about the total demand in delivery period. Producers/Consumers are worried about decreasing/increasing of the total demand. Hence, since scenarios are ordered in an demand increasing manner, i.e. $c_{s+1}^{td} \geq c_s^{td}$, the PDF of total demand scenarios from their viewpoint is right-skewed/left-skewed. Using Beta PDF we can model left and right skewness of PDFs easily. Beta PDF is formulated as below [19]:

$$p(x) = \frac{(x-v)^{\tau-1}(w-x)^{\beta-1}}{B(\tau,\beta) * (w-v)^{\beta-\tau}} \tag{1}$$

$$B(\tau,\beta) = \int_0^1 x^{\tau-1}(1-x)^{1-\beta}dx \tag{2}$$

where x is a random variable and $v \leq x \leq w$. Beta PDF is right/left skewed if $\tau \geq \beta/\tau \leq \beta$. For each producer i/consumer j at hour t of day d we have $\sum_{s\in S} \rho_{i,s}^{\pi,td} = 1/\sum_{s\in S} \rho_{j,s}^{u,td} = 1$. Fig. 1 compares probabilities of concern scenarios from the view point of a producer and a consumer about the variation of demand function intercept or actually the demand of an arbitrary consumer j for a specific hour in delivery period. As Fig. 1 shows, since the producers/consumers are worried about decreasing/increasing the demand in delivery period, PDF of concern scenarios from the viewpoint of producers/consumers have right/left-skewness. It should be noted that the reality that will happen in future is unique and can be modeled by Normal PDF or any other distribution. However, in the proposed method probabilities of scenarios are affected based on the concerns of each market player about happening of each scenario.

2.4. Market players' actions and contracts and electricity market settlement procedure

Producers can participate in forward contract and day-ahead electricity markets strategically. However, since consumers need to provide their loads, it is not possible for them to participate in both markets strategically. Consumers can behave strategically in forward contract since if their load is not provided in forward market they have another option to provide it, but in order to provide their demand, they participate in day-ahead electricity market at delivery period non-strategically and as price takers.

Forward contracts are settled similar to settlement of a CfD [1]. To this end, producers/consumers bid/offer in day-ahead electricity market such that each producer/consumer is dispatched at least equal to its forward contract quantity. They receive/pay MCP from/to ISO for the whole dispatched power in day-ahead electricity market. Then, contract parties settle their forward contracts by exchanging the difference between electricity market price and forward contract price for each MWh of forward contract.

Linear supply function model is used to model the behavior of producers and consumers in both forward and day-ahead markets. It is assumed that slope of bid/offer of a producer/consumer is equal to the slope of its marginal cost/marginal utility function. The intercept of the submitted bid/offer function of a producer/consumer is his or her decision variable when he or she behaves strategically [20].

In forward contract negotiation process, each producer submits different bids to different consumers, and each consumer submits different offers to different producers. Each contract party games on the intercepts of his/her bid/offer function to reach the optimal values for price and quantities for forward contract between each producer and each consumer. Price and quantity of each forward contract is determined from the intersection of contract parties bid and offer functions.

In day-ahead electricity market, at hour t of day d and scenario s, each producer i submits two bids to ISO: (1) a low price bid for his/her total forward contracts to guaranty his/her obligations in forward market, i.e. M_i^c and (2) a strategic bid function to sell the rest of his/her generation capacity i.e. $\alpha_{i,s}^{e,td} + b_i^e Q_{i,s}^{e,td}$. Each consumers also submits two offers to ISO: (1) a high price offer for his/her total forward contracts to guaranty his/her obligations in forward market, i.e. N_j^c and (2) a non-strategic affine utility function to buy the rest of their required demand that is not supplied through forward contracts, i.e. $c'^{td}_{j,s} - d_j^e Q_{j,s}^{e,td}$ in which $c'^{td}_{j,s} = c_{j,s}^{td} - d_j^e \sum_{i\in F} Q_{ji}^c$. Fig. 2 illustrates this procedure. ($/MWh)

It is assumed that the transmission system is not congested in different operation conditions and hence, transmission system constraints are not considered in the model. This will also decrease the computation burden significantly.

3. Nash equilibrium of the joint forward and day-ahead electricity markets

In this section, the proposed model for computing Nash equilibrium of the joint forward and day-ahead electricity markets is formulated. To

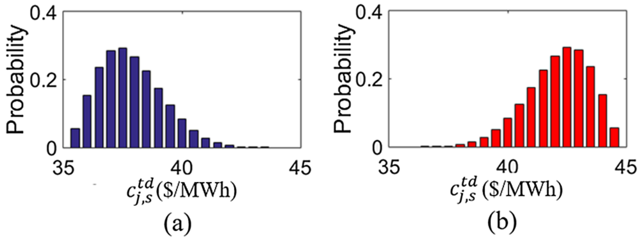


Fig. 1. (a)/(b) represents the probabilities of demand concern scenarios form the viewpoint of a producer/consumer for hour t of day d, respectively.

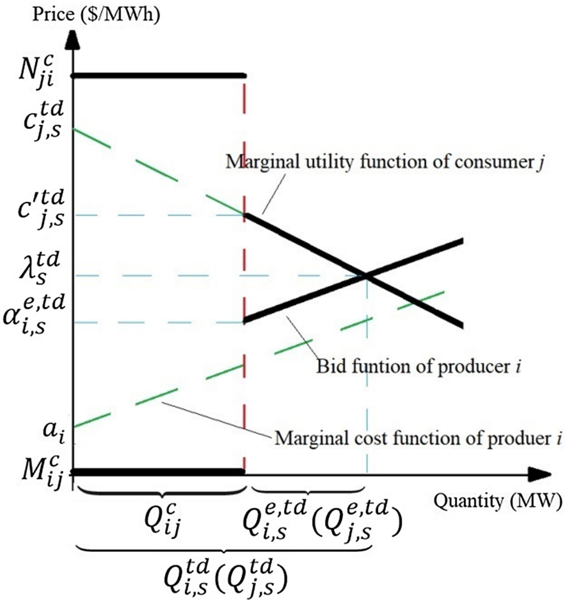


Fig. 2. Bids of producers and offers of consumers in day-ahead electricity market.

this end, forward market and day-ahead market operation are formulated. Then, profit of producers and consumers are formulated, and finally, SFE calculation procedure is explained and modeled.

3.1. Forward contract market modeling

Each producer/consumer has the option to conclude contracts with every consumer/producer in forward contract market. Assume the proposed bid function of producer i to consumer j for the whole delivery period is $F_{ij}^{c} = \alpha_{ij}^{c} + b_i^{c} Q_{ij}^{c}$ and the proposed offer of consumer j to producer i is $F_{ji}^{c} = \epsilon_{ji}^{c} - d_j^{c} Q_{ji}^{c}$. Forward contract price and quantity between producer i and consumer j is obtained by finding the intersection of bid/offer functions as follows:

$$Q_{ji}^{c} = Q_{ij}^{c} = \frac{\epsilon_{ji}^{c} - \alpha_{ij}^{c}}{b_i^{c} + d_j^{c}} \tag{3}$$

$$F_{ji}^{c} = F_{ij}^{c} = \frac{b_i^{c}\,\epsilon_{ji}^{c} + d_j^{c}\alpha_{ij}^{c}}{b_i^{c} + d_j^{c}} \tag{4}$$

In fact, the accepted contract point is where both producer and consumer agree on the same price and quantity for their forward contract. Using (3) and (4) forward contract prices and quantities between producer i and consumer j are modeled as functions of decision variables of producers and consumers, i.e. α_{ij}^{c} and ϵ_{ji}^{c}.

3.2. Day-ahead electricity market operation modeling

ISO receives the bids of producers and offers of consumers and determines the MCP and quantities of powers that each producer is committed to generate or each consumer is allowed to consume such that the social welfare maximizes. ISO optimization is performed for each hour of each day in delivery period and each concern scenario. Bids M_{ij}^{c} and N_{ji}^{c} are independent from the forward contract prices that just guaranty wining the proposed contract power in the day-ahead electricity market. All the proposed power with these bids will be accepted by ISO and hence, it is not necessary to consider them in the social welfare maximization problem. The second part of the optimization involves the optimal supply function bids of producers for their remained capacity after participation in forward market and utility functions of the consumers for providing the rest of their demand that is not provided in forward market. In the rest of this paper, phrase "dispatched power of producer i/consumer j in day-ahead electricity market" represent the dispatched power of producer i in electricity market due to the second part of its bid/offer and it is shown with $Q_{i,s}^{e,td}/Q_{j,s}^{e,td}$, and phrase "total scheduled power of producer i/consumer j in day-ahead electricity market" represent the sum of dispatched power of producer i/consumer j in day-ahead electricity market due to the first and second parts of its bid/offer and it is shown with. $Q_{i,s}^{td}/Q_{j,s}^{td}$.

The ISO social welfare maximization problem at hour t of day d and concern scenario s can be formulated as below:

$$\max \quad SW_{t,d,s} = \sum_{j\in J} \left(c'^{td}_{j,s} Q_{j,s}^{e,td} - \frac{1}{2} d_j^{e} Q_{j,s}^{e,td2}\right) - \sum_{i\in F} \left(\alpha_{i,s}^{e,td} Q_{i,s}^{e,td} + \frac{1}{2} b_i^{e} Q_{i,s}^{e,td2}\right) \tag{5}$$

$$s.\ t. \qquad \sum_{i\in F} Q_{i,s}^{e,td} - \sum_{j\in J} Q_{j,s}^{e,td} = 0,\ \lambda_s^{td} \tag{6}$$

where λ_s^{td} is Lagrangian multiplier for constraint (6) and represents the MCP at hour t of day d and scenario s. Optimization (5)–(6) is a convex optimization problem. Hence, optimal solution can be found by writing the KKT optimality conditions of the optimization problem. Writing the KKT optimality conditions of the problem for all hours, days and scenarios and rearranging the formulas similar to the proposed method in [21] yields:

$$Q_{i,s}^{e,td} = \sum_{j\in J} \frac{c_{j,s}^{td}}{Bb_i^{e} d_j^{e}} - \sum_{j\in D} \sum_{u\in F} \frac{Q_{ju}^{c,}}{Bb_i^{e}} + \sum_{u\in F} m_u^{i} \alpha_{u,s}^{e,td} \qquad \forall \\ t\in T,\ d\in D,\ i\in F,\ s\in S \tag{7}$$

$$Q_{j,s}^{e,td} = \frac{-1}{d_j^{e}} \sum_{l\in J} Z_l^{j} c_{ls}^{td} + \sum_{l\in J} \sum_{i\in F} Z_l^{j} Q_{li}^{c} - \sum_{u\in F} \frac{\alpha_{u,s}^{e,td}}{Bd_j^{e} b_u^{e}} \qquad \forall \\ t\in T,\ d\in D,\ j\in J,\ s\in S \tag{8}$$

$$\lambda_s^{td} = \frac{1}{B}\left(\sum_{j\in J} \frac{c_{j,s}^{td}}{d_j^{e}} - \sum_{j\in J} \sum_{i\in F} Q_{ji}^{c} + \sum_{u\in F} \frac{\alpha_{u,s}^{e,td}}{b_u^{e}}\right) \qquad \forall \\ t\in T,\ d\in D,\ s\in S \tag{9}$$

where:

$$B = \sum_{i\in F} \frac{1}{b_i^{e}} + \sum_{j\in J} \frac{1}{d_j^{e}}, \tag{10}$$

$$m_u^{i} = \begin{cases} \frac{1}{Bb_i^{e} b_u^{e}} & i \neq u \\ \frac{1}{Bb_u^{e}}\left(\frac{1}{b_u^{e}} - B\right) & i = u \end{cases} \qquad \forall\ i,\ u\in F \tag{11}$$

$$Z_l^{j} = \begin{cases} \frac{1}{Bd_l^{e}} & j \neq l \\ \frac{1}{Bd_l^{e}} - 1 & j = l \end{cases} \qquad \forall\ j,\ l\in J \tag{12}$$

In fact, variables $Q_{i,s}^{e,td}$, $Q_{j,s}^{e,td}$ and $\lambda_s^{td}$are represented as functions of decision variables of producers i.e. $\alpha_{i,s}^{e,td}$ and quantity of forward contracts powers i.e. Q_{ij}^{c}.

3.3. Formulation of producers' profit

Each producer tries to maximize its profit in forward contracts and day-ahead electricity markets over different concern scenarios of the delivery period. Expected profit of producers can be formulated as follows:

$$Max \quad E(\pi_i) = \sum_{d\in D}\sum_{t\in T}\sum_{s\in S}\rho_{i,s}^{\pi,td}\left[\lambda_s^{td}Q_{i,s}^{e,td} + \sum_{j\in J}F_{ij}^{c}Q_{ij}^{c} - a_i\left(Q_{i,s}^{e,td} + \sum_{j\in J}Q_{ij}^{c}\right) - \frac{1}{2}b_i^{e}\left(Q_{i,s}^{e,td} + \sum_{j\in J}Q_{ij}^{c}\right)^2\right] \tag{13}$$

$$s.\,t. \qquad Q_{i,s}^{e,td} + \sum_{j\in J}Q_{ij}^{c} \le \bar{Q}_i \perp \bar{\mu}_{i,s}^{td} \ge 0 \qquad \forall\, t\in T,\, d\in D,\, s\in S \tag{14}$$

$$-(Q_{i,s}^{e,td} + \sum_{j\in J}Q_{ij}^{c}) \le -\underline{Q}_i \perp \underline{\mu}_{i,s}^{td} \ge 0 \qquad \forall\, t\in T,\, d\in D,\, s\in S \tag{15}$$

$$Q_{i,s}^{e,(t+1)d} - Q_{i,s}^{e,td} \le Q_i^{up} \perp \mu_{i,s}^{up,td} \ge 0 \qquad \forall\, t\in T,\, d\in D,\, s\in S \tag{16}$$

$$Q_{i,s}^{e,(t-1)d} - Q_{i,s}^{e,td} \le Q_i^{down} \perp \mu_{i,s}^{do,td} \ge 0 \qquad \forall\, t\in T,\, d\in D,\, s\in S \tag{17}$$

$$Q_{i,s}^{e,td} \ge 0 \perp \mu_{i,s}^{e,td} \ge 0 \qquad \forall\, t\in T,\, d\in D,\, s\in S \tag{18}$$

$$Q_{ij}^{c} \ge 0 \perp \mu_{ij}^{c} \ge 0 \qquad \forall\, j\in J \tag{19}$$

The first and second terms of objective function (13) represent the revenue of producer i from day-ahead electricity market and forward contracts with different consumers, respectively. The remained terms of objective function (13) represent the total cost of the producer for generating power in both forward and day-ahead electricity markets. Constraints (14)–(15) limit the output power of producer's power plants to their maximum and minimum values, respectively. Constraints (16)–(17) are ramp-rate limits of producer i and constraints (18)–(19) guaranty day-ahead market power and forward contract power of producer i are positive.

3.4. Formulation of consumers' profit

Expected profit of consumer j is calculated by subtracting the actual value of electricity for that consumer form its payment to ISO and producers. Hence, the Expected profit optimization problem of consumer j over different concern scenarios of the delivery period is formulated as follows:

$$\max \qquad \forall\, E(U_j) = \sum_{d\in D}\sum_{t\in T}\sum_{s\in S}\rho_{j,s}^{u,td}\left[c_{j,s}^{td}\left(Q_{j,s}^{e,td} + \sum_{i\in I}Q_{ji}^{c}\right) - \frac{1}{2}d_j^{e}\left(Q_{j,s}^{e,td} + \sum_{i\in I}Q_{ji}^{c}\right)^2 - \lambda_s^{td}Q_{j,s}^{e,td} - \sum_{i\in F}F_{ji}^{c}Q_{ji}^{c}\right] \tag{20}$$

$$s.\,t \qquad \forall\, Q_{j,s}^{e,td} \ge 0 \perp \mu_{j,s}^{e,td} \qquad \forall\, t\in T,\, d\in D,\, s\in S \tag{21}$$

$$Q_{ji}^{c} \ge 0 \perp \mu_{ji}^{c} \qquad \forall\, i\in F \tag{22}$$

The first two terms of objective function (20) represent the total utility of consumer j for supplying his/her demand from both forward and day-ahead electricity markets. The last two terms of objective function (20) represent the sum of the consumer's payments to ISO for buying power from day-ahead electricity market and to different producers for buying power from forward market, respectively. Constraints (21)–(22) guaranty the positivity of day-ahead market power and forward contract power quantities.

3.5. Nash equilibrium of the joint forward contract and day-ahead market

In Nash equilibrium of a market none of the producers and consumers can increase his/her profit by changing his/her strategy unilaterally. Different methods can be applied for finding the equilibrium in problems with only producers, only consumers and both producers and consumers as market players [20–22]. In this paper, in order to find the SFE of the model, optimization problems of producers and consumers should be solved considering forward contracts and day-ahead electricity markets outcomes. This turns the profit optimization problem of each producer/consumer into a bi-level optimization problem. Profit optimization problems of producers/consumers, i.e. (13)–(19)/(20)–(22), are the outer-level problems. Forward market problem, i.e. (3)–(4), and ISO optimization problem, i.e. (5)–(6), are two inner-level problems of each outer-level problem. The outer-level problems are coupled and form an EPEC. Solution of the EPEC is SFE. One of the well-known approaches to solve an EPEC is to add KKT optimality conditions of inner-level problems as constraints to the outer-level problem and solve the KKT conditions of all outer-level problems using nonlinear programming [20,23].

Eqs. (3)–(4) and (7)–(9) represent the KKT optimally conditions of inner problems and formulate variables F_{ij}^{c}, Q_{ij}^{c}, $Q_{j,s}^{e,td}$, $Q_{j,s}^{e,td}$and λ_s^{td}, as functions of α_{ij}^{c}, $\alpha_{i,s}^{e,td}$ and ϵ_{ji}^{c}. So, in order to solve the EPEC, it is enough to add Eqs. (3), (4), (7) and (9) as equality constraint to producers optimization problem (13)–(19) and Eqs. (3), (4), (8) and (9) as equality constraint to consumers optimization problem (20)–(22) and solving the KKT optimally conditions of the upgraded outer-level optimization problems of producers and consumers. Before solving the KKT optimally conditions for these problems, it is proposed to substitute variables F_{ij}^{c}, Q_{ij}^{c}, $Q_j^{e,ts}$, $Q_j^{e,ts}$ and λ^{ts} from (3)–(4) and (7)–(9) into (13)–(22) instead of adding them as constraints. This reduces the computation burden and simplifies the model by reducing the number of constraints and variables. Following this procedure, profit optimization problems will be rearranged as functions of decision making variables of producers and consumers, i.e. $\alpha_{ij}^{c}\ \forall\, i\in F, j\in J$, $\alpha_{i,s}^{e,td}\ \forall\, i\in F, d\in D, t\in T, s\in S$ and $\epsilon_{ji}^{c}\ \forall\, j\in J, i\in F$.

In order to find the KKT optimally conditions of upgraded outer-level optimization problems of producers and consumers, lagrangian function of upgraded optimization problems of each producer and consumer, i.e. $L(\pi_i)$ and $L(U_j)$, should be formed, respectively. Then, KKT optimally conditions are [24] are (1) Derivations of $L(\pi_i)$ with respect to producers' decision making variables, i.e. α_{ij}^{c}, $\alpha_{i,s}^{e,td}$, for each producer, and derivations of $L(U_j)$ with respect to consumers' decision making variables, i.e. ϵ_{ji}^{c}, (2) Upgraded inequality constraints, (3) Positivity of lagrangian multipliers, and (4) Complementary slackness conditions between each inequality constraint and its corresponding lagrangian multiplier.

Details of deriving KKT conditions are presented in Appendix A. Eqs. (23)–(33) of Appendix A are sets of KKT conditions of all producers and consumers that turn optimization problems of all producers and consumers into a nonlinear complementarity problem. PATH solver in GAMS software that uses a Newton-based method is applied to find the optimal solution of the proposed model.

4. Case study

In this section, the proposed model is applied to a test system and simulation results are analyzed. Test system is a power system with 3 producers and 5 consumers. Each of these producers can refer to a set of generation units that belong to one firm and are simplified as one producer with one large-scale power plant [19]. Producers' cost function data are presented in Table 1. Each consumer can also presents a set of loads with one aggregated utility function. Delivery period of each day is 8 h. For the sake of simplicity, hourly demand in different days of the delivery period is assumed to be similar and hence, the problem is solved for only one day. Consumers utility function data are presented in Table 2 and Fig. 3.

15 concern scenarios are identified around the intercept of utility function of each consumer for each hour of delivery period. These

Table 1
Producers cost function parameters.

	Producer number		
	1	2	3
a_i(\$/MW h)	16	5.6	20
b_i^c, b_i^e(\$/MW^2 h)	0.007	0.026	0.017
Q_i^{up}(GW/h)	0.4	0.5	0.2
Q_i^{down}(GW/h)	0.4	0.5	0.2
$\bar{Q}_i$(GW)	5	5	3
$\underline{Q}_i$ (GW)	0.5	0.7	0.2

Table 2
Consumers utility function parameters.

	Consumer number				
	1	2	3	4	5
d_j^c, d_j^e(\$/MW^2 h)	0.005	0.003	0.007	0.0004	0.0004

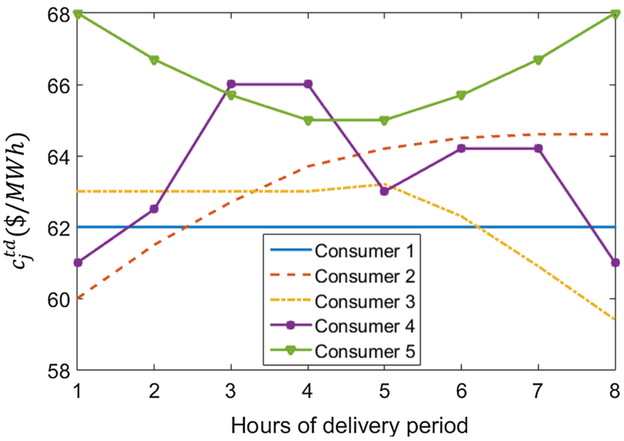


Fig. 3. Intercepts of utility functions of consumers in different hours of delivery period.

scenarios are unique from the viewpoint of both producers and consumers but probability of each scenario may be different from the viewpoint of different producers and consumers. Values of $c_{j,s}^{e,td}$ in different concern scenarios are uniformly distributed between $0.9c_j^{e,td}$and $1.1c_j^{e,td}$. Fig. 3 shows values of $c_{j,s}^{e,td}$ of different consumers over the delivery period. In fact, $c_j^{e,td}$is the basic value of utility function intercept for determining $c_{j,s}^{e,td}$. Probability of concern scenarios are modeled using Beta PDF. Beta PDF parameters is assumed to be the same for different hours of all days of delivery period and different for each market player. Tables 3 and 4 show the sets of (τ, β) parameters for determining the probabilities of concern scenarios for both producers and consumers, i.e. $\rho_{i,s}^{\pi,td}$ and $\rho_{j,s}^{u,td}$, respectively. Fig. 4 compares the probabilities of these scenarios for different producers and consumers. As Fig. 4 shows, producer 2 is less concern about the demand in the delivery period while producer 3 is more concern than the other producers. It also shows that consumers 2 and 3 are less concern than the other consumer about the future while consumer 1 has the highest concern. Described power system in Tables 1–4 and Figs. 3 and 4 is referred to as “base case”.

4.1. Simulation results of the base case

Simulation results for the base case is presented in Tables 5–7 and Figs. 5–7. Table 5 shows the power quantities of forward contracts between each producer and consumer in the contract period, respectively. As Table 5 shows, the share of producer 1 and consumer 5 from the contracted powers in forward market are more than other producers and consumers. These results are influenced by different factors. Producer 1 is one of the low cost producers with high generation capacity and this helps him/her to bid such that he/she can sell large value of power both in forward and day-ahead markets. Producer 1 also has high concerns about the future and this forces this producer to sell more quantities of power in forward market than day-ahead electricity market. As it is shown in Fig. 3, consumer 5 has the highest value of demand request compared to other consumers. Moreover, as Fig. 4 shows, consumer 5 is highly concerned about the future which forces him/her to contract more quantity of power than other consumers in forward market.

Table 6 compares the ratio of concluded forward contract of each player to its expected dispatched power in day-ahead market. This ratio is affected by two factors, concerns of market players about the future and variation in demand or output power of each market player. Since the contracts are settled by physical delivery, the quantity of forward contracts is limited by minimum scheduled power of market players in different hours and scenarios. As the variation of demand or output power of a market player gets lower, the contracted power can get closer to expected total scheduled power and the ratio increases. Producer 3 and consumer 1 have the highest concerns and lower variation in total scheduled power compared to other market players and consequently the highest ratios.

In Table 7, Weighted contract price for each producer is calculated by $F_i^c = \sum_{j\in D} Q_{ij}^c F_{ij}^c / \sum_{j\in D} Q_{ij}^c$ and for each consumer is calculated by $F_j^c = \sum_{i\in F} Q_{ji}^c F_{ji}^c / \sum_{i\in F} Q_{ji}^c$. Total weighted contract price is calculated by $F^c = \sum_{i\in F} \sum_{j\in D} Q_{ij}^c F_{ij}^c / \sum_{i\in F} \sum_{j\in D} Q_{ij}^c$ and is equal to 53.97 \$/MWh. Forward contract prices are different in different contracts but the difference between the highest and lowest contract price is 4.9% of the total weighted contract price. Table 7 shows that the producer 3/consumer 1 which has the highest concerns about future is willing to contract with low/high prices to sell/buy more power by forward contracts and reduces the risk of undesirable market price variation in delivery period. producer 2/consumer 2 which has low concerns about the future tries to benefit from forward contracts and sell//buy power with highest/lowest possible prices.

Simulation results of day-ahead electricity market for different hours of delivery period and different scenarios are presented in Figs. 5–7. For the sake of comparison, total contracted power of each consumer or producer is inserted in each figure using a dark blue flat plane. Fig. 5 shows the variation in the total scheduled power of consumers and total demand in different hours of the delivery period and different concern scenarios. As it is expected, for each consumer, the demand in different hours follows a trend similar to the trend of that consumer’s utility function intercept. As the concerns about future increases the gap between total contracted power and minimum demand in different hours and scenarios decreases. For a specific delivery hour, total scheduled power of consumers increases as scenario number increases. This happens because, demand uncertainty in delivery period is modeled by changing the intercept of the consumers’ utility functions and their load increase as scenario number increases. Hence, the scheduled power increases as scenario number increases.

Fig. 6 shows the variations of total scheduled power of producers in

Table 3
Producers’ beta PDF parameters.

	Producer number		
	1	2	3
(τ, β)	(1.7,8)	(4,7)	(1.2,7.1)

Table 4
Consumers' beta PDF parameters.

	Consumer number				
	1	2	3	4	5
(τ, β)	(7.5,1.2)	(4,4)	(5.1,4.8)	(8,2)	(7,1.9)

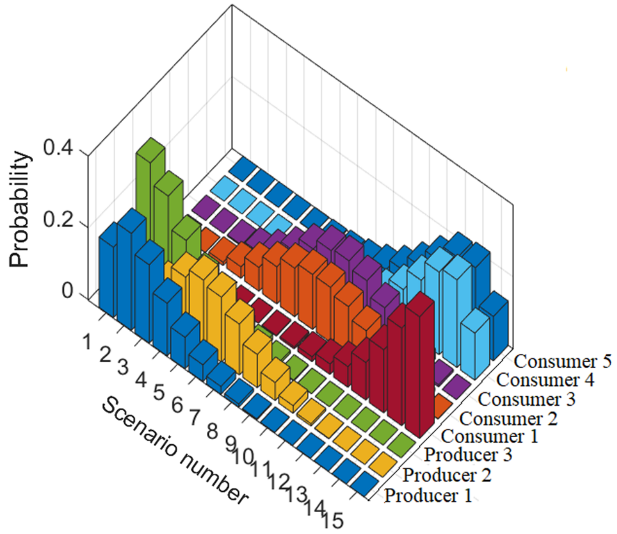


Fig. 4. Comparing the probabilities of concern scenarios for different producers and consumers.

Table 5
Forward contract quantities between different producers and consumers.

Q^{c}_{ij}(MW)	Consumers					Total contract
	1	2	3	4	5	
Producer 1	558.1	558.7	256.1	648.6	801.7	2822
Producer 2	147.6	119.2	44.3	167.1	223	701
Producer 3	318.9	295.1	162.8	354.9	434.9	1566
Total contract	1025	972.4	463.2	1170.7	1458.9	

Table 6
Ratio of total forward contract power of each market player to its expected dispatched power in day-ahead market.

Producers			Consumer				
1	2	3	1	2	3	4	5
1.39	0.62	5.69	4.42	0.67	1.06	1.48	1.16

Table 7
Forward contract prices between different producers and consumers.

F^{c}_{ij}($/MW h)	Consumers					Weighted price
	1	2	3	4	5	
Producer 1	54.62	53.51	53.62	54.43	55.04	54.38
Producer 2	54.69	54.31	54.27	54.62	54.85	54.64
Producer 3	53.11	52.40	52.66	52.93	53.26	52.93
Weighted price	54.16	53.27	53.34	54.01	54.48	

day-ahead electricity market. Total scheduled power of producers mainly follows a trend similar to the trend of the total demand in different hours and scenarios. Producer 1 which its production cost is lower than other producers, plays a more important rule on supplying the demand and reaches to its maximum generation capacity in most hours and scenarios.

Variations of MCP in different hours and scenarios is compared with total weighted contract price is drawn in Fig. 7. As Fig. 7 shows producers are more worried about occurring the first 7 scenarios in which the MCP decreases and consumers are more worried about occurring the last 7 scenarios in which MCP increases. This different concerns about future lead to forward contract prices that are greater than MCP in most of the first 7 scenarios and help the producers to hedge themselves against the risk of decreasing the MCP. Different concerns about future also lead to forward contract prices that are lower than MCP in the last 7 scenarios and help the consumers to hedge themselves against the risk of increasing MCP. Expected value for MCP is 56 $/MWh which is about 4 percent greater that total weighted contract price.

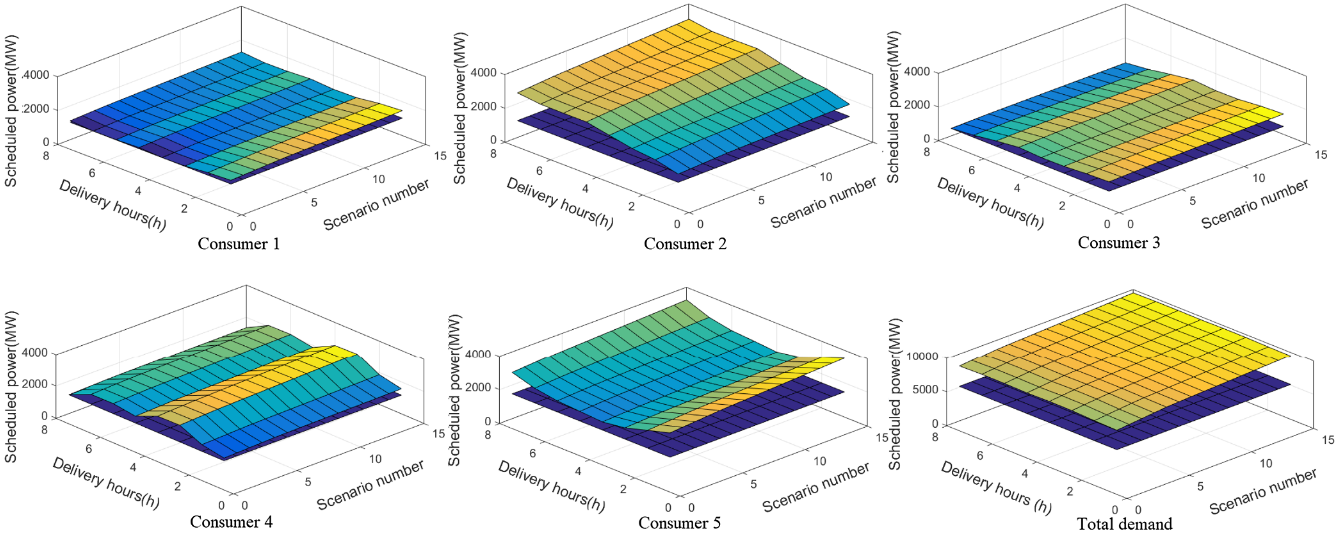
4.2. Effects of forward contracts on the profits of market players and MCP

In this subsection impacts of considering forward contract market parallel with day-ahead electricity market are studied. To this end, simulation results for the cases that there is and there is not a forward market are compared. In order to model the case that forward market is not considered in the model, Eqs. (23) and (26)–(29) should be solved together while the forward contract market equations are omitted from the ISO operation and producers and consumers profits formulas. Expected profits of market players from their viewpoint for both cases are compared in Table 8. In normal situation, it is expected that increase in the profit of some market players leads to decrease in the profit of some others. But as Table 8 shows, the expected profits of all producers and consumers has increased after considering forward market in the power system. This happens because the expected values of profits in Table 7 are calculated using concern scenarios. In fact the expected profits are calculated based on the concerns of the market players and contains their risks management preferences. So, it can be said that in aggregate of profit and risk both producers and consumers benefit form forward contracts. In more detail, for instance, for consumer 1 if the probability of different scenarios are assumed to be equal, the profit increases 59% while when the expected profit is calculated by concern scenarios the profit increases 172.8%. This difference represents the improvement in risk management concerns of consumer 1. Table 8 also shows that the increase in the profit of consumers is more than increase in the profit of producers. This is because of strategic behavior of consumers in forward contract market which let them to increase their profit by strategical bidding in contracting procedure while in day-ahead market consumers are price taker players. But since the producers have already been strategic market players in day-ahead market their profit improvement is not as much as consumers.

Comparing the result in different scenarios and hours shows that MCP in the base case is about 0.24% lower than the MCP in the case that there is no forward contract market.

4.3. Comparing the proposed risk management method with CVaR method

In order to evaluate the efficiency of applying concern scenarios for modeling the risk management preferences of the market players, the proposed method in this paper is compared with the case that CVaR is used for risk management. Modeling the problem with CVaR has complications which is beyond the scope of this paper. Hence, the problem is modeled for a simple case in which there is only one producer and one consumer in the system. It is assumed that the producer behaves strategically in both forward and day-ahead markets, while the consumer is price taker in both forward and day-ahead markets. It is also assumed that delivery period is an hour of a day and hence, the ramp-rate constraints are removed. In order to model this problem using concern scenarios, it is enough to solve (23), (24), (26), (27), (30) and (31) assuming there is only one producer and one consumer. In this case, since the consumer is assumed to be price taker in both markets, variable ϵ^{c}_{ji} turns into a fixed parameter and it is assumed to be equal to



Consumer 1 Consumer 2 Consumer 3

Consumer 4 Consumer 5 Total demand

Fig. 5. Forward contract power quantities and day-ahead scheduled power of consumers in different hours and scenarios.

the expected value of $c_{j,s}^{td}$ over different forecast scenarios. Formulation of this simple problem with CVaR is presented in Appendix B. Market players are producer 1 and consumer 1 which are introduced in Tables 1–4 and Fig. 3. It should be noted that forecast scenarios that are used in CVaR method are modeled by Normal PDF but concern scenarios that are used in concern scenario method are modeled using Beta PDF similar to Fig. 4.

Fig. 8 shows how the profit of the producer 1 changes in different forecast scenarios for different values of parameters β in concern scenario method and parameter γ in CVaR method. Parameter τ is assumed to equal to 2. As Fig. 8 shows, by choosing proper values for β in concern scenario method, the results are exactly similar to the results of the CVaR method. It should be noted that in concern scenario method, it is not required to define new variables, add any other part to the objective function, or add new constraints. This reduces the complexity and run time of the problem, specially, for large-sale problems.

4.4. Impacts of different concerns of market players on the results

Different concerns of producers or consumers affect their strategies in both markets. In order to study the impacts of changing the concerns of market players on the results parameter τ of Beta PDF for producer 2 and consumer 2 are varied between 1.5 and 6.5 while the values of parameter β are fixed and equal to their values in Tables 3 and 4. Increasing parameter τ for a producer/consumer means reducing/growing his/her concerns about the future. Total contracted power, expected dispatched power in day-ahead market, and total scheduled power of the producer 2 and consumer 2 for different values of parameter τ are presented in Fig. 9.

As Fig. 9 confirms by growing the concerns of producer 2 and consumer 2 about the future, they try to trade more power through forward contracts and less power through day-ahead market to hedge themselves against the risks of undesirable market price variations.

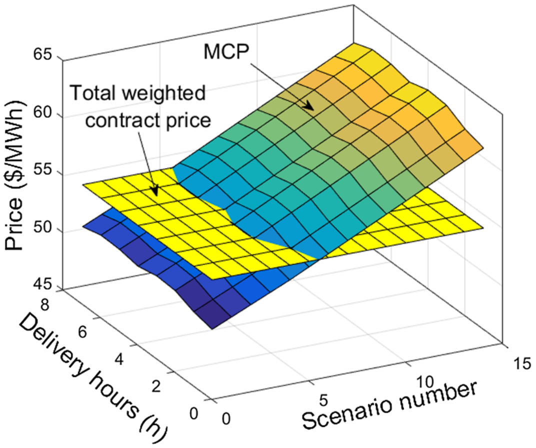
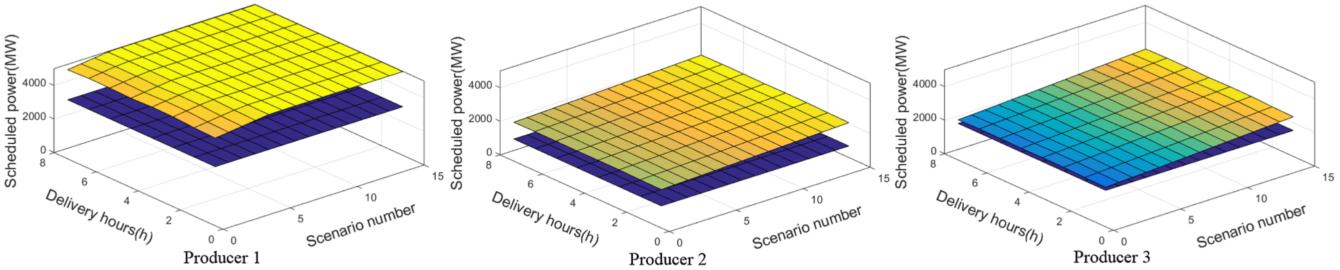


Fig. 7. Comparing the total weighted contract price with MCP in different hours and scenarios.

4.5. Impacts of increasing the uncertainty of demand in delivery period

Uncertainty of demand is modeled by defining some scenarios around parameters c_j^{td}. In order to model the increase in uncertainty, the range of variations of parameters $c_{j,s}^{td}$in different scenarios increases. To this end, in the defined scenarios in the base casec_j^{td} is varied from $(1-\omega)c_j^{td}$ to $(1+\omega)c_j^{td}$, while number of scenarios and probability of each scenario are kept constant. In this way, domain of variation of demand can be changed by changing the value of parameter ω. Table 9 compares the results for the base case ($\omega = 0.1$) and the case that ω



Producer 1 Producer 2 Producer 3

Fig. 6. Total scheduled power of producers in different hours and scenarios.

Table 8
Comparing the profits of market players in the base case with the case that there is no forward contract.

	Without forward contracts	With forward contracts	% of increase in the profits
Producer 1	737,130	790,290	7.2
Producer 2	362,330	363,930	0.44
Producer 3	237,400	251,850	6.08
Consumer 1	31,069	84,750	172.8
Consumer 2	71,265	97,190	36.4
Consumer 3	23,210	35,730	53.91
Consumer 4	62,798	119,250	89.88
Consumer 5	119,060	182,180	53.01

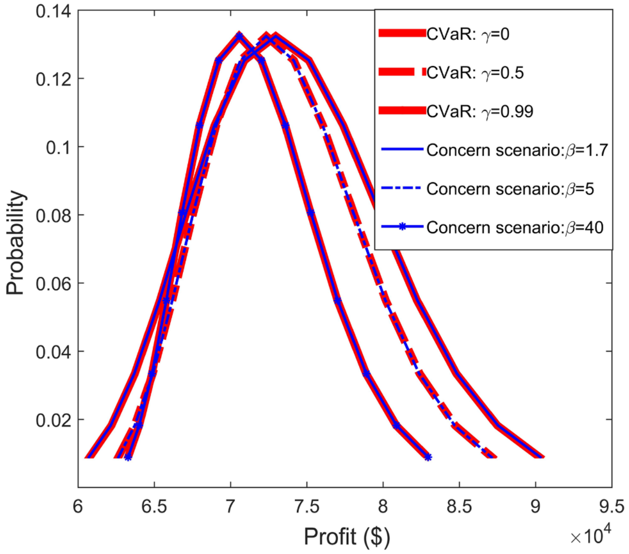
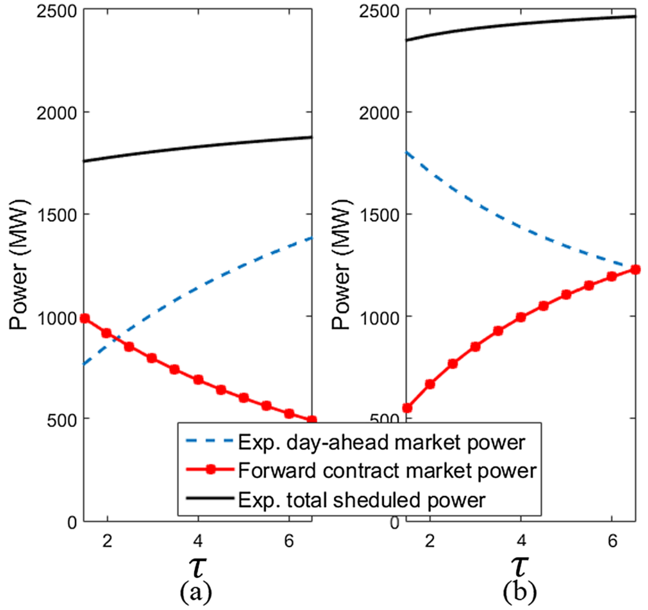


Fig. 8. Comparing the concern scenario method and CVaR method results.



(a)

(b)

Fig. 9. Variations of scheduled power in forward and day-ahead markets by changing the value of τ for (a) producer 2 and (b) consumer 2.

Table 9
Impacts of increasing the uncertainty on the simulation results.

		$\omega = 0.1$	$\omega = 0.15$
Producer 1	Forward contract power	2822	3230.3
	Exp. Day-ahead power	2032.8	1423.6
Consumer 5	Forward contract power	1458	1835.2
	Exp. day-ahead power	1257.1	941.3

increases to 0.15 for producer 1 and consumer 5. As Table 9 shows by increasing the uncertainty both producer and consumer behave such that they contract more power in forward market and hedge themselves against the risks associated with uncertainties in day-ahead market at the delivery period. By 0.05 increase in ω, (50% increase in uncertainty) the share of forward contract of the total expected scheduled power for producer 1 and consumer 5 increases form 58.1% to 69.4% and from 53.7% to 66.1%, respectively.

4.6. Impacts of increasing the bargaining power of consumers on the results

In previous simulations it is assumed that the slope of the bid functions in both forward and day-ahead markets are equal to the slope of the associated marginal cost and marginal utility functions. Time of concluding forward contract is usually far from the delivery period and consumers have the opportunity to conclude other forward contracts or provide their required electric energy from the day-ahead market if electricity price is high in forward market. However, time of participating in day-ahead market is close to the delivery period and consumer have no other choice to provide the rest of their required power. Hence, they have to buy the rest of their required load from day-ahead market even if electricity price is high. Hence, price elasticity of the consumers in forward market is more than their price elasticity in day-ahead market. In fact, consumer have more bargaining power in forward market than day-ahead market. In order to show the effect of more elastic nature of consumers in forward market on the results, it is assumed that one of the consumers participates in forward market with a bid function that its slope is equal to 70% of the slope of its utility function or its slope in day-ahead market while slopes of bids of other consumers in both forward and day-ahead markets are equal to the slope of their utility functions similar to the base case. Then the simulation results are assessed. This procedure is repeated for all consumers. Table 10 compares the weighted contract prices, quantities and profits of consumers in this case with the base case. As Table 10 shows by increasing the bargaining power of each consumer, he/she is able to contract more quantity of power with lower price that increases his/her profit between 4.35 and 6.64%. Comparing the results with the results in Fig. 5 indicates that the amount of increase in total contracted power depends on the gap between total quantity of forward contracts and minimum total scheduled power in day-ahead market for each consumer in the base case. In fact, the constraint of physical delivery restricts the increase in the total contracted power for consumers 1, 3 and 4.

4.7. Impacts of producers' contracting obligations

In this subsection, it is assumed that one or some producers are obligated to sell at least a minimum quantity of their output capacity though forward contracts. In order to model this situation, it is enough

Table 10
Impacts of increasing the bargaining power of consumers on the producers.

% of variation of:	Consumers				
	1	2	3	4	5
Q_j^c	0.001	6.62	0.03	0.02	8.14
Weighted F_j^c	−1.12	−0.59	−0.77	−1.06	−1.03
Exp. Profit	5.93	4.36	4.35	4.47	6.64

Table 11
Percentage of variations in forward contract quantities between different producers and consumers considering contract obligations.

	Consumers				
	1	2	3	4	5
Producer 1	−6.34	−0.89	−12.46	−5.78	−0.55
Producer 2	36.5	59.16	112.66	33.58	30.60
Producer 3	−6.06	−0.85	−11.45	−5.54	−0.541

Table 12
Percentage of variations in forward contract prices between different producers and consumers considering contract obligations.

	Consumers				
	1	2	3	4	5
Producer 1	−0.31	−0.02	−0.41	−0.27	−0.02
Producer 2	−3.33	−3.46	−3.21	−3.42	−3.31
Producer 3	−0.17	−0.01	−0.24	−0.14	−0.01

to at add constraint $\sum_{j\in J} Q_{ji}^{c} \geq Q_i^{c,min}$ to optimization problems (13)–(19) of each producer i that is obligated to sell at least $Q_i^{c,min}$ MW trough forward contracts. In order to study the impacts of this obligation, it is assumed that producer 2 is obligated to sell at least 1 GW trough forward contracts. Percentage of variations in the forward contract quantities and prices for all market players compared to the base case are presented in Tables 11 and 12, respectively. As tables 11 and 12 show, contract obligations reduce the bargaining power of producer 2 and force him/her to bid lower prices to gain more forward contract quantities. This behavior not only reduces the contract prices of producer 2 but also reduces the contract prices and quantities of the other producers and consequently decreases producers' profits. Expected profits of producers 1 and 3 reduces about 0.5% and profit of producer 2 reduces by 3.6%. Profits of all consumers increases between 3.2% and 4.2% due to contracting more powers in lower prices.

4.8. Real size test system simulation results

In order to show the ability of model for applying to real case problems, a test system with triple size of the studied system in Subsection 4.1 is considered in this subsection. In this case, the number of producers in the test system is equal to 9, which is equal to the number of power plants in Khorasan electricity network in Iran [25]. The number of consumers also are increased to 15. Cost/Utility functions parameters of the first 3 producers/5 consumers are the same as cost/utility functions parameters of producers/consumer in Table 2/Table 3 and Fig. 3. Cost/Utility functions of the second 3 producers/5 consumers have intercepts equal to the intercepts of first 3 producers/5 consumers and slopes equal to 0.9 of the slopes of the first 3 producers/5 consumers. Finally, cost/utility functions of third 3 producers/5 consumers have intercepts equal to 0.9 of the intercepts of the first 3 producers/5 consumers and slopes equal to the slopes of the first 3 producers/5 consumers. Same procedure is repeated for parameters τ and β. Simulation results of scheduled powers for forward contracts and day-ahead market for different market players are presented in Tables 13 and 14. Total power is distributed among different market players in the form of forward contracts and day-ahead market dispatch based on their cost and utility functions parameters and concerns about the future. Expected value for day-ahead MCP is 52.57 $/MWh and total weighted price of forward contracts is 48.53 $/MWh. This result follows the trend of the base case results in which the contract prices obtained lower than expected day-ahead market price.

5. Conclusion

In this paper, a comprehensive SFE model for studying the mutual impacts of forward contracts and day-ahead markets is proposed. Proposed model considers the strategic behaviour of both producers and consumers in determination of price and quantities of forward contracts. Market players are also able to trade the rest of their supply or demand through a day-ahead market. The proposed SFE model helps to study how day-ahead market prices changes after concluding forward contracts, determine the range of variations of forward contract prices, and understand the share of forward contracts and day-ahead market for covering the demand in different situations. A new risk management method is also presented, inspired by the different concerns of market players about the future day-ahead market prices. Simulation show that exact CVaR and the presented method yield the same results, whereas the presented method is much easier, needs less variables and constraints, and hence, it is much faster than CVaR method.

Simulation results show that both producers and consumers forward market would benefit from contact market considering the aggregation of profit and risk. Forward contracts increase the profit of consumers more than the profit of producers. Day-ahead electricity market prices reduces slightly after considering forward contacts. Contract prices for different contracts are different but close to each other and their difference is less than 4.6%. Expected value of the MCP from market operator's view point is about 4% greater than aggregated forward contracts price for understudy test system. Simulation results also shows as the concerns of market players about the future increases, or as the variations of the generation or consumption in different hours of

Table 13
Producers' simulation results of test system with 9 producers and 15 consumers.

Producer Number	1	2	3	4	5	6	7	8	9
Total Contracted power (MW)	4327	1217	1579	4703	1031	1744	4545	1483	1603
Exp. Day-ahead power (MW)	717	2527	233	595	3036	268	631	2356	232
Total Exp. scheduled power (MW)	5044	3744	1812	5298	4067	2012	5176	3839	1835

Table 14
Consumers' simulation results of test system with 9 producers and 15 consumers.

Consumer Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Total Contracted power (MW)	1541	2365	846	1820	2565	1712	2627	940	2022	2850	487	640	113	516	1184
Exp. Day-ahead power (MW)	342	1171	533	894	877	380	1301	593	993	975	156	789	377	600	612
Total Exp. consumed power (MW)	1883	3536	1379	2714	3442	2092	3928	1533	3015	3815	643	1429	490	1116	1796

delivery period decreases, the quantity of contracted power in forward market increases. Fifty percent growth in the uncertainty about future demand increases the share of forward contract power quantities up to 22%. Simulation results also shows that if one or more producers are forced to sell their power through forward contacts, profits of these producers and even other producers decrease and consequently profits of consumers increase due to reduction in forward contract prices.

Considering transmission system constraints and modeling other different types of producers and consumers like renewable energy resources and shiftable loads are future directions of this study.

Appendix A. KKT optimally conditions

In this appendix, KKT optimally conditions of all producers and consumers are presented. For each producer i, derivations of $L(\pi_i)$ with respect to α_{ij}^{c}, $\alpha_{i,s}^{e,td}$ are as below:

$$\begin{aligned}\frac{\partial L(\pi_i)}{\partial \alpha_{i,s}^{e,td}} = {} & \rho_{i,s}^{\pi,td}\left[-\sum_{u\in F}\left(m_u^i+\frac{1}{Bb_u^e}m_i^i\right)\alpha_{u,s}^{e,td}+\left(\frac{1}{Bb_i^e}\right)^2\sum_{j\in J}\sum_{u\in F}\frac{\epsilon_{ju}^c-\alpha_{ij}^c}{b_u^c+d_j^c}+b_i^e m_i^i\sum_{j\in J}\frac{\epsilon_{ju}^c-\alpha_{ij}^c}{b_i^c+d_j^c}-\left(\frac{1}{Bb_i^e}\right)^2\sum_{j\in J}\frac{c_{j,s}^{td}}{d_j^e}+m_i^i a_i\right]-m_i^i\\ & \mu_{i,s}^{e,td}+m_i^i(\bar{\mu}_{i,s}^{td}-\underline{\mu}_{i,s}^{td})+m_i^i(\mu_{i,s}^{up,(t-1)d}-\mu_{i,s}^{up,td}+\mu_{i,s}^{do,(t+1)d}-\mu_{i,s}^{do,td})=0 \qquad \forall\, i\in F,\, t\in T,\, d\in D,\, s\in S\end{aligned} \tag{23}$$

$$\begin{aligned}\frac{\partial L(\pi_i)}{\partial \alpha_{ij}^{c}} \\ = {} & \sum_{d\in D}\sum_{t\in T}\sum_{s\in S}\rho_{i,s}^{\pi,td}\left[-\sum_{u\in F}\left(\frac{1}{B^2b_u^eb_i^e}+b_i^e m_u^i\right)\alpha_{u,s}^{e,td}+\frac{1}{Bb_i^e}\left(\frac{1}{B}+b_i^e\right)\sum_{l\in J}\sum_{u\in F}\frac{\epsilon_{lu}^c-\alpha_{ul}^c}{b_u^c+d_l^c}+\left(\frac{1}{B}-b_i^e\right)\sum_{l\in J}\frac{\epsilon_{li}^c-\alpha_{il}^c}{b_i^c+d_l^c}-d_j^c\right.\\ & \left.\frac{\epsilon_{ji}^c-\alpha_{ij}^c}{b_i^c+d_j^c}+\frac{b_i^c\,\epsilon_{ji}^c+d_j^c\alpha_{ij}^c}{b_i^c+d_j^c}-\frac{1}{Bb_i^e}\left(\frac{1}{B}+b_i^e\right)\sum_{l\in J}\frac{c_{l,s}^{td}-\mu_{l,s}^{td}}{d_l^e}+(\frac{1}{Bb_i^e}-1)a_i\right]-\frac{1}{Bb_i^e}\sum_{d\in D}\sum_{t\in T}\sum_{s\in S}\mu_{i,s}^{e,td}+\sum_{D\in F}\sum_{t\in T}\sum_{s\in S}\left(\frac{1}{Bb_i^e}-1\right)(\bar{\mu}_{i,s}^{td}-\underline{\mu}_{i,s}^{td})+\mu_{ij}^c\\ = {} & 0 \qquad \forall\, i\in F, j\in J\end{aligned} \tag{24}$$

Upgraded inequality constraints, positivity of lagrangian multipliers and complementary slackness conditions for producers are presented as below:

$$\frac{1}{Bb_i^e}\sum_{j\in J}\frac{c_{j,s}^{td}}{d_j^e}-\frac{1}{Bb_i^e}\sum_{j\in J}\sum_{u\in F}\frac{\epsilon_{ju}^c-\alpha_{uj}^c}{b_u^c+d_j^c}+\sum_{u\in F}m_u^i\alpha_{u,s}^{e,td}+\sum_{j\in J}\frac{\epsilon_{ji}^c-\alpha_{ij}^c}{b_i^c+d_j^c}-\bar{Q}_i\le 0\perp\bar{\mu}_{i,s}^{td}\ge 0 \qquad \forall\, i\in F,\, t\in T,\, d\in D,\, s\in S \tag{25}$$

$$\frac{-1}{Bb_i^e}\sum_{j\in J}\frac{c_{j,s}^{td}}{d_j^e}+\frac{1}{Bb_i^e}\sum_{l\in J}\sum_{u\in F}\frac{\epsilon_{lu}^c-\alpha_{ul}^c}{b_u^c+d_l^c}-\sum_{u\in F}m_u^i\alpha_{u,s}^{e,td}-\sum_{j\in J}\frac{\epsilon_{ji}^c-\alpha_{ij}^c}{b_i^c+d_j^c}+\underline{Q}_i\le 0\perp\underline{\mu}_{i,s}^{td}\ge 0 \qquad \forall\, i\in F,\, t\in T,\, d\in D,\, s\in S \tag{26}$$

$$\frac{1}{Bb_i^e}\sum_{j\in J}\frac{c_{j,s}^{(t+1)d}-c_{j,s}^{td}}{d_j^e}+\sum_{u\in F}m_u^i(\alpha_{u,s}^{e,(t+1)d}-\alpha_{u,s}^{e,td})-Q_i^{up}\le 0\perp\mu_{i,s}^{up,td}\ge 0 \qquad \forall\, i\in F,\, t\in T,\, d\in D,\, s\in S \tag{27}$$

$$\frac{1}{Bb_i^e}\sum_{j\in J}\frac{c_{j,s}^{(t-1)d}-c_{j,s}^{td}}{d_j^e}+\sum_{u\in F}m_u^i(\alpha_{u,s}^{e,(t-1)d}-\alpha_{u,s}^{e,td})-Q_i^{down}\le 0\perp\mu_{i,s}^{do,td}\ge 0 \qquad \forall\, i\in Ft\in T,\, d\in D,\, s\in S \tag{28}$$

$$\frac{-1}{Bb_i^e}\sum_{j\in J}\frac{c_{j,s}^{td}}{d_j^e}+\frac{1}{Bb_i^e}\sum_{j\in J}\sum_{u\in F}\frac{\epsilon_{ju}^c-\alpha_{uj}^c}{b_u^c+d_j^c}-\sum_{u\in F}m_u^i\alpha_{u,s}^{e,td}\le 0\perp\mu_{i,s}^{e,td}\ge 0 \qquad \forall\, i\in F,\, t\in T,\, d\in D,\, s\in S \tag{29}$$

$$-\frac{\epsilon_{ji}^c-\alpha_{ij}^c}{b_i^c+d_j^c}\le 0\perp\mu_{ij}^c\ge 0 \qquad \forall\, i\in F, j\in J \tag{30}$$

For each consumer j, derivations of $L(U_j)$ with respect to ϵ_{ji}^c is calculated as follows:

$$\begin{aligned}\frac{\partial L(U_j)}{\partial\, \epsilon_{ji}^{c}} \\ = {} & \sum_{d\in D}\sum_{t\in T}\sum_{s\in S}\rho_{j,s}^{u,td}\left[\frac{1}{Bd_j^e}\sum_{u\in F}\frac{1}{b_u^e}\left(\frac{1}{B}-d_j^e\right)(\alpha_{u,s}^{e,td})-\frac{Z_j^j}{B}\sum_{l\in J}\sum_{u\in F}\frac{\epsilon_{lu}^c-\alpha_{ul}^c}{b_u^c+d_l^c}+\frac{Z_j^j}{B}\sum_{l\in J}\frac{c_{l,s}^{td}}{d_l^e}+d_j^e(1+Z_j^j)\sum_{u\in F}\frac{\epsilon_{ju}^c-\alpha_{uj}^c}{b_u^c+d_j^c}+b_i^c\right.\\ & \left.\frac{\epsilon_{ji}^c-\alpha_{ij}^c}{b_i^c+d_j^c}+\frac{b_i^c\,\epsilon_{ji}^c+d_j^c\alpha_{ij}^c}{b_i^c+d_j^c}-c_{j,s}^{e,td}(1+Z_j^j)\right]-Z_j^j\sum_{d\in D}\sum_{t\in T}\sum_{s\in S}\mu_{j,s}^{e,td}-\mu_{ji}^c=0 \qquad \forall\, i\in F, j\in J\end{aligned} \tag{31}$$

Finally, upgraded inequality constraints, positivity of lagrangian multipliers and complementary slackness conditions for consumers are presented as below:

$$\frac{1}{d_j^e}\sum_{l\in J}Z_l^j c_{ls}^{td}-\sum_{l\in J}\sum_{i\in F}Z_l^j Q_{li}^c+\sum_{u\in F}\frac{\alpha_{u,s}^{e,td}}{Bd_j^e b_u^e}\le 0\perp\mu_{j,s}^{e,td}\ge 0 \qquad \forall\, j\in J,\, t\in T,\, d\in D,\, s\in S \tag{32}$$

$$-\frac{\epsilon_{ji}^c-\alpha_{ij}^c}{b_i^c+d_j^c}\le 0\perp\mu_{ji}^c\ge 0 \qquad \forall\, i\in F, j\in J \tag{33}$$

Appendix B. Formulation of the problem with CVaR

In this Appendix, optimal gaming of one producer in forward and day-ahead markets is modeled. CVaR method is used for risk management. Forward and day-ahead markets equations are obtained using (3)–(4) and (7)–(9) considering there is only one producer and one consumer in the system. Since, the problem is solved for an hour of a day, indices t and d and ramp-rate constraints are removed. Moreover, since it is assumed that the consumer is price taker in this problem, variable ϵ_{11}^{c} turns into a fixed parameter equal to the expected value of $c_{1,s}$ over different forecast scenarios. Profit maximization problem of producer 1 considering CVaR is formulated as below [6]:

$$MaxE(\pi_1) = \sum_{s\in S} \rho_s^{\pi}\left[\lambda_s Q_{1,s}^{e} + F_{11}^{c} Q_{11}^{c} - a_1(Q_{1,s}^{e} + Q_{11}^{c}) - \frac{1}{2} b_1^{e}(Q_{1,s}^{e} + Q_{11}^{c})^2\right] - \gamma\left[\zeta - \frac{1}{1-\theta}\sum_{s\in S}\rho_s^{\pi}\eta_s\right] \tag{34}$$

$$s.\ t. \qquad \forall\ Q_{1,s}^{e} + Q_{11}^{c} \le \bar{Q}_i \qquad \forall\ s \in S \tag{35}$$

$$-(Q_{1,s}^{e} + Q_{11}^{c}) \le -\underline{Q}_i \qquad \forall\ s \in S \tag{36}$$

$$Q_{1,s}^{e} \ge 0 \qquad \forall\ s \in S \tag{37}$$

$$Q_{11}^{c} \ge 0 \tag{38}$$

$$Q_{11}^{c} = \frac{\epsilon_{11}^{c} - \alpha_{11}^{c}}{b_1^{c} + d_1^{c}} \tag{39}$$

$$F_{11}^{c} = \frac{b_1^{c}\ \epsilon_{11}^{c} + d_1^{c}\alpha_{11}^{c}}{b_1^{c} + d_1^{c}} \tag{40}$$

$$Q_{1,s}^{e} = \frac{c_{1,s}}{Bb_1^{e} d_1^{e}} - \frac{Q_{11}^{c,}}{Bb_1^{e}} + m_1^{1}\alpha_{1,s}^{e} \qquad \forall\ s \in S \tag{41}$$

$$\lambda_s = \frac{1}{B}\left(\frac{c_{1,s}}{Bb_1^{e} d_1^{e}} - \frac{Q_{11}^{c,}}{Bb_1^{e}} + \frac{\alpha_{1,s}^{e}}{b_1^{e}}\right) \qquad \forall\ s \in S \tag{42}$$

$$-\left[\lambda_s Q_{1,s}^{e} + F_{11}^{c} Q_{11}^{c} - a_1(Q_{1,s}^{e} + Q_{11}^{c}) - \frac{1}{2} b_1^{e}(Q_{1,s}^{e} + Q_{11}^{c})^2\right] + \zeta - \eta_s \le 0 \qquad \forall\ s \in S \tag{43}$$

$$\eta_s \ge 0 \qquad \forall\ s \in S \tag{44}$$

Problem (34)–(44) is a quadratic optimization problem and convex. By solving this problem, optimal values of $\alpha_{1,s}^{e}$ and α_{11}^{c} and consequently optimal values of Q_{11}^{c}, $Q_{1,s}^{e}$, F_{11}^{c} and λ_s are determined. Parameter γ is the weighting factor that determines the importance of risk management for producer 1.

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