Put Option Pricing and Its Effects on Day-Ahead Electricity Markets

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***Abstract*—In this paper, the impacts of strike and premium prices of put option contracts on put option and day-ahead electricity markets are studied. To this end, first a comprehensive equilibrium model for a joint put option and day-ahead markets is presented. Interaction between put option and day-ahead markets, uncertainty in demand, and elasticity of consumers to strike price, premium price, and day-ahead price are taken into account in this model. Then, a formula for computing strike prices at which producers and consumers are willing to trade put option is presented and a new method for put option pricing is proposed. By applying the presented model to a test system, the interaction between the put option and day-ahead markets is studied.**

***Index Terms*—Cournot competition, equilibrium of joint put option and day-ahead markets, option market modeling, put option pricing.**

I. Introduction

FINANCIAL electricity markets have been developed beside the physical electricity markets during the last two decades. Power producers sell a portion of their electric energy in financial electricity markets in order to hedge themselves against quantity and price risks [1]. In electricity markets, short time uncertainties of power producers are covered by attending in intraday market, whereas long time uncertainties are hedge by attending in financial markets [2], [3]. A financial electricity market may affect the strategies of producers in the related physical electricity market and consequently the electricity price of the physical market that is used as reference price in the financial market [4], profits of participants, and social welfare. Physical market regulators are interested to find out how and to what extent a financial electricity market affects the related physical electricity market [5]. To this end, financial and physical electricity markets must be modeled and studied in detail to clarify the impacts of interaction between these markets.

A. Basic Concepts

Financial derivatives such as forwards, options, and futures are employed in financial electricity markets in order to manage and reallocate risks. An option provides more flexibility than a forward or a future contract since holder of an option has the right to exercise it depending on the availability of his/her generating units and the pool price behavior [6]. Option contracts are divided into call and put options. A call/put option is a contract that gives the buyer (the owner) the right, but not the obligation, to buy/sell an underlying asset at a specified price on or before a specified date [7]. Buyer of a call/put option contract must pay a price to the seller for this right that is called premium price. The specified price in which the electricity can be bought/sold by call/put option holder up to the expiration date is called strike price. In option markets, strike price is usually determined by financial market operator before it can be traded. Buyer and seller of an option contract must agree on a premium price at the time of concluding option contract. If the owner exercises the call/put option, the seller of the call/put option contract has the corresponding obligation to fulfill the transaction, that is, to sell/buy the underlying commodity. In some option markets, option contracts can be settled by physical delivery or by cash, but in some option markets option contracts can be settled only by cash [7].

In order to describe the option exercising method, consider a producer that has bought a put option from a consumer. The producer participates in day-ahead market to sell his/her whole generation power including the power that has been sold in the option market. The consumer also participates in day-ahead market to buy the required electric energy. Independent system operator (ISO) determines market clearing price (MCP) and dispatched power of each producer and consumer for each hour of scheduling period. At each hour, the consumer pays MCP to ISO and ISO pays MCP to the producer for every dispatched mega watt. If MCP is greater than the strike price of the option, the option is not exercised. This means the consumer pays MCP and the producer receives MCP for every dispatched mega watt in day-ahead market. If MCP is less than the strike price of the option, the option is exercised. In this case, the consumer pays difference of the strike price and MCP to the producer for every mega watt of the option contract in addition to paying MCP to ISO for every dispatched mega watt in day-ahead market. This means the consumer pays the strike price and the producer receives the strike price for the every mega watt of the option contract.

B. Literature Review

Impacts of option contracts on the bidding strategies of physical market participants are studied in [6], [8]–[13]. To hedge price risks of risk-averse producers and consumers, an optimal strategy for selecting optional forward contracts is presented in [8]. In order to hedge the quantity risks of a load serving entity in a physical competitive market, the optimal bidding strategy of the load serving entity for buying forward and option contracts are determined in [9]. An option market beside a physical electricity market is considered in [10] and an approach for calculating optimal strike price from the viewpoint of a market maker is proposed. In [11] and [6], a multistage stochastic model is presented to determine the optimal strategies of a risk-averse producer in forward, option and pool markets considering price

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and generation availability risks. Bezerra *at al.* [12] develop a stochastic optimization model for determining the bidding strategy of a producer in an energy call option auction. In this auction, bidders can offer both premium and strike prices. An integrated risk management framework for strategic trading of a producer in spot, option, and fuel markets is proposed in [13].

The equilibrium of both physical and option markets are studied in [5], [14]–[18]. In [14], a new forward contract with bilateral options is introduced in order to hedge the price volatility risks of buyers and sellers in the physical market. In [15], a two-period equilibrium model for financial and physical electricity markets is presented. In [15], strategic producers compete with their rivals by setting their supply functions in spot market and by setting their generation power in financial option market. In [16], effects of put and call option contracts on the strategies of producers on a physical market with Cournot competition is studied. The influence of call option contracts on the equilibrium of joint spot and option markets is studied in [17]. Ruiqing *et al.* [17] consider a Cournot model for spot market. Bessembinder and Lemmon [18] focus on hedging decisions and wholesale electricity market equilibrium considering forward contracts. They consider both forward market prices and output decisions as endogenous variables. In [18], forward price is calculated based on the statistical relationship between historical data of physical market price and forward price. The impacts of day-ahead energy pricing on the operation of the joint option and day-ahead electricity markets are studied in [19]. In [20], the impacts of premium price limits on the option and day-ahead electricity markets are studied from the viewpoint of market regulator.

In this paper, the impacts of strike and premium prices of put option contracts on put option and day-ahead markets are studied from the viewpoint of market regulator. For this study, we need to know strategic behavior of producers, i.e., concluded option contract of each producer at option market and its bid at day-ahead market. However, strategic behaviors of producers are unknown, depend on their interactions, and change in different situations in oligopoly markets. In order to overcome this problem and take into account the interactions of market participants, it is assumed that the understudy put option and day-ahead markets have reached to their Nash equilibrium, and the impacts are studied at the Nash equilibrium of the joint option and day-ahead markets. Although real electricity markets do not work on their Nash equilibrium, mature electricity markets work near their Nash equilibrium and Nash equilibrium models are widely used for the study of electricity markets when interaction of producers should be considered [19]–[27]. Nash equilibrium is a point in which no producer can increase its pay off by changing its strategy unilaterally [19]–[24], [28], [29]. Nash equilibrium is computed by solving coupled optimizations of producers all together. Since all optimization problems are solved together, profit of each producer is maximized considering the strategic behavior of other producers [19], [20], [24], [25], [30], [31].

The difference between this paper and the available research works is that this paper considers detail of financial derivatives contracts, and interaction between financial and physical electricity markets.

The main contributions of this paper are as follows:

1) presenting an equilibrium model for a joint put option and day-ahead markets;
2) proposing a formula for computing strike prices at which producers and consumers are willing to trade put option; and
3) presenting a method for put option pricing.

Contributions 2 and 3 are very useful in the operation of joint option and day-ahead markets [32].

A few methods for electricity option pricing are presented based on the historical data of electricity spot markets [33], [34]. The proposed model in this paper can be used as an option pricing method that can consider the strategic behavior of producers and the impacts of new installed transmission lines, power plants, or other facilities that will start operation in trading period.

The rest of this paper is organized as follows. In Section II, an equilibrium model for a joint put option and day-ahead markets is presented and a formula for computing strike prices is proposed. By applying the presented model to a four-producer power system, a method for put option pricing is presented in Section III. Concluding remarks are provided in Section IV.

II. Modeling Joint Put Option and Day-Ahead Markets

Consider a power system with a physical day-ahead electricity market and a financial electricity market including an option market. Producers and consumers can hedge themselves against risks of price volatility due to demand uncertainty by making derivative contracts in the option market. Put and call option contracts are two different derivative instruments and are traded independently. Here, we focus on European put option contracts as an independent hedging tool. Hereafter, option is used instead of put option in this paper for the sake of simplicity.

Option contracts for a specified delivery day are traded before the associated day-ahead market. The values of strike and premium prices of option contracts affect the strategies of producers in the option market and consequently affect the volume of concluded option contracts. The volume of concluded option contracts affects the strategies of producers in the day-ahead market, and consequently the day-ahead electricity price. In turn, day-ahead prices affect the strategies of producers in the option market. Hence, day-ahead and option markets can mutually affect each other. The main goal of this paper is to determine the impacts of premium and strike prices of option contracts on the strategies of producers in the day-ahead and option markets.

A. Markets Structure and Decision Framework

The understudy physical electricity market is an oligopoly day-ahead market with POOLCO structure and Cournot competition [16], [17], [25]. In practice, almost all electricity markets have Bertrand or supply function competition. However, since solutions of Cournot equilibrium model and supply function equilibrium model are the same [35], Cournot equilibrium model has been widely used for electricity market modeling [16], [17], [21], [25]. Load is uncertain with normal distribution and elastic with constant elasticity. However, consumers are not strategic.

The understudy financial electricity market is a put option market with physical delivery [11], [36]. Participants of a physical electricity market can trade standard put option contracts in the associated option market. Each standard put option contract has a specific mega watt size, a specified strike price, and a specified delivery period [32], [36]. Strike prices and delivery periods of standard option contracts are determined by the related financial market operator [32], [36]. A put option trader chooses the desired option contract based on the desired delivery period and the desired strike price, and then offers the required mega watt size and a suitable premium price to buy or sell it [32], [36]. If bids of a seller and a buyer are matched, the deal

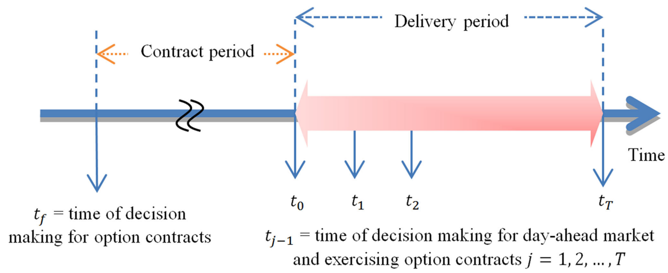

Fig. 1. Timeline for decision making of producers in the option and day-ahead markets.

is done [32], [36]. In delivery period, whenever the day-ahead market price is less than the strike price of the contracted option, buyers of put option contract exercise their right to sell the contracted mega watt at the contracted strike price [36].

Although put option and day-ahead markets are operated independently, participation of power producers and consumers in both markets connects these markets together, especially if the put option market has physical delivery as EEX market [36]. If the put option market has physical delivery, strategic behavior of power producers in the put option market affects residual load in the day-ahead market and consequently the strategic behavior of power producers in the day-ahead market, and in turn, the day-ahead market price. Change in day-ahead market price may affect the strategic behavior of participants in the put option market.

Financial and physical market operators are independent. However, usually there is a market regulator or a supervisory board that regulates financial and physical electricity markets [36]. In this paper, the impacts of strike and premium prices of put option market on the day-ahead and put option markets are studied from the viewpoint of this market regulator or supervisory board.

Delivery period of an option contract usually consists of 24 hours or specified hours of a specified week, month, season, or year. Without loss of generality, it is assumed that delivery period consists of specified hours of several consecutive days. These hours are referred to as *study hours*. Hours of delivery period are numerated with t_j, where $j = 1, 2, \ldots, T$. Timeline for producers' decision making in the option and day-ahead markets is shown in Fig. 1. Consider a delivery period. Producers should make the following decisions optimally to maximize their profits over this delivery period.

1) Several months before starting the delivery period, each producer should decide about the volume of the option contract that should buy from the option market for this delivery period [32], [36]. Suppose producer i buys Q_i^O mega watt option contract from the option market at t_f, as shown in Fig. 1.
2) One day before each days of the delivery period, each producer should decide how much power it should generate at each study hour of the next day in the day-ahead market [25]. Suppose producer i generates Q_{it}^{Dh} mega watt at hour t of the delivery period.
3) One day before each days of the delivery period after clearing day-ahead market, each producer should decide what portion of its option contract must be exercised at each study hour of the next delivery day [32], [36]. It is assumed that producer i exercises Q_{it}^O mega watt of its total option contract, i.e., Q_i^O, at hour t of the delivery period. Here, it is assumed that the exercised volume of option contracts is a continuous variable. In real world, it may be a discrete variable.

In order to consider uncertainty in demand, S scenarios are defined for demand over the delivery period based on distribution functions of loads at different hours of the delivery period. Inverse demand function at hour t of scenario s can be written as follows:

$$\lambda_{st} = N_{st} - \gamma Q_{st}^L \quad t = t_1, t_2, \ldots, t_T \tag{1}$$

where λ_{st} and Q_{st}^L are electricity price and total network load at hour t of scenario s, respectively. N_{st} and γ are coefficients of inverse demand function at hour t of scenario s in \$/MWh and \$/MW^{2}h, respectively. Generation cost of producer i at hour t of scenario s is given as follows:

$$C_i(Q_{ist}^O + Q_{ist}^{Dh}) = a_i(Q_{ist}^O + Q_{ist}^{Dh}) + \frac{1}{2} b_i (Q_{ist}^O + Q_{ist}^{Dh})^2 \tag{2}$$

where a_i and b_i are cost function coefficients of producer i in \$/MWh and \$/MW^{2}h, respectively. In this paper, first in Section II-B it is assumed that transmission network has no constraint to avoid the impact of congestion on the simulation results and an equilibrium model for the joint option and day-ahead markets is presented. After presenting a formula for option contract area in Section II-C, transmission network is considered in the proposed equilibrium model in Section II-D.

B. No-Transmission Model

Each strike price can be considered as a specified commodity. Here, it is assumed that only a strike price has been considered for the understudy delivery period. Participating in option market is not mandatory. Hence, producers can be categorized in two sets A and B. Set A consists of producers that attend in both option and day-ahead markets. Set B consists of producers that only attend in the day-ahead market. Producer i of set A determines its strategy so that its total expected profit from option and day-ahead markets is maximized. Therefore, the optimization problem of this producer is formulated as follows:

$$\begin{aligned} \max_{Q_{ist}^O, Q_{ist}^{Dh}, Q_i^O, f_{iK}} \quad & E(\pi_i) \\ = \sum_{s=1}^{S} \sum_{t=t_0}^{t_T} p_s \Bigg(& Q_{ist}^O K + Q_{ist}^{Dh} \lambda_{st} - \Big(a_i (Q_{ist}^O + Q_{ist}^{Dh}) \\ & + \frac{1}{2} b_i (Q_{ist}^O + Q_{ist}^{Dh})^2 \Big) \Bigg) - Q_i^O T f_{iK} e^{r T_C} \end{aligned} \tag{3}$$

s.t. :

$$(Q_{ist}^O + Q_{ist}^{Dh}) \le \overline{Q_i} \quad \forall s \in \Omega \quad \forall t \in \mathcal{T} : \mu_{ist} \tag{4}$$

$$Q_{ist}^O \le Q_i^O \; \forall s \in \Omega \quad \forall t \in \mathcal{T} : \omega_{ist} \tag{5}$$

$$\begin{aligned} \lambda_{st} = N_{st} - \gamma \left(\sum_{m \in A} (Q_{mst}^O + Q_{mst}^{Dh}) + \sum_{v \in B} Q_{vst}^{Dh} \right) \\ \forall s \in \Omega \quad \forall t \in \mathcal{T} : \theta_{st} \end{aligned} \tag{6}$$

$$\begin{aligned} K - f_{iK} e^{r T_C} \le N_{st} - \gamma \sum_{m \in A} Q_m^O \quad \forall s \in \Omega \\ \forall t \in \mathcal{T} : \beta_{ist} \end{aligned} \tag{7}$$

$$Q_{ist}^O \ge 0, Q_{ist}^{Dh} \ge 0, Q_i^O \ge 0, f_{iK} \ge 0 \quad \forall s \in \Omega \quad \forall t \in \mathcal{T} \tag{8}$$

where K is the strike price of option contract and f_{iK} is the premium bid of producer i in \$/MWh. In (3)–(8), r is interest rate, T_C is contract period or time to delivery in year and is equal to $t_0 - t_f$, $\overline{Q_i}$ is the maximum generation capacity of producer i in mega watt, $\mathcal{T}$ is the set of hours of delivery period, Ω is the set of scenarios of demand, p_s is the probability of scenario s, μ_{ist} is the dual variable of upper capacity limit of producer i at hour t of scenario s, ω_{ist} is the dual variable of upper capacity limit for exercising option contract of producer i at hour t of scenario s, and β_{ist} is the dual variable of the constraint that is enforced by elasticity of load to premium bid of producer i in the option market at hour t of scenario s.

The first term of the objective function (3) denotes the expected income of producer i from the exercising option contracts at different hours of the delivery period. The second term of (3) denotes the expected income of producer i from the physical day-ahead market over the delivery period. The sum of third to sixth terms of (3), which are located inside parenthesis, indicates the total expected generation cost of producer i over the delivery period. The last term of (3) denotes the cost of buying option contract.

Decision making about option exercising by producer i at hour t of scenario s is modeled by maximizing $(Q^O_{ist}K + Q^{Dh}_{ist}\lambda_{st})$ in the objective function, considering the fact that demand function is constant at hour t of scenario s. If strike price K is greater than day-ahead market price λ_{st}, the profit of producer i at scenario s is maximized if $Q^O_{ist}K$ is maximized, i.e., if Q^O_{ist} is equal to Q^O_i, or if producer i exercises its option contract. If strike price K is smaller than λ_{st}, the profit of producer i is maximized if $Q^{Dh}_{ist}\lambda_{st}$ is maximized, i.e., if Q^O_{ist} is equal to zero or if the producer i does not exercise its option contract.

Inequalities (4) enforce upper generation limit of producer i to every hour of every scenario of the delivery period (EHESDP). Inequalities (5) impose the upper exercising limit of option contract of producer i to EHESDP. Constraints (6) express the relationship between electricity price and total consumption at EHESDP. Constraints (7) model the elasticity of load to strike price and premium bid of producer i at EHESDP. According to constraints (7), consumers purchase electricity from option market as long as total payment for 1 MWh electricity is less than their willingness to pay at EHESDP. Willingness to pay of consumers at EHESDP is specified by the related demand function. Since producer i of set A attends both in the option and day-ahead markets, decision variables of its optimization problem are Q^O_i, Q^O_{ist}, f_{iK}, and Q^{Dh}_{ist}, $\forall s \in \Omega, \forall t \in \mathcal{T}$.

The Karush–Kuhn–Tucker (KKT) optimality conditions of each producer i of set A are as follows:

$$\left\{p_s\left(-K + \gamma Q^{Dh}_{ist} + a_i + b_i(Q^O_{ist} + Q^{Dh}_{ist})\right) + \omega_{ist} + \mu_{ist} \geq 0\right\} \perp Q^O_{ist} \geq 0 \;\; \forall s \in \Omega \quad \forall t \in \mathcal{T} \tag{9}$$

$$\left\{p_s\left(-N_{st} + \gamma\left(\sum_{m \in A}(Q^O_{mst} + Q^{Dh}_{mst}) + \sum_{v \in B} Q^{Dh}_{vst}\right) + \gamma Q^{Dh}_{ist} + a_i + b_i(Q^O_{ist} + Q^{Dh}_{ist})\right) + \mu_{ist} \geq 0\right\} \perp Q^{Dh}_{ist} \geq 0 \quad \forall s \in \Omega \quad \forall t \in \mathcal{T} \tag{10}$$

$$\left\{T f_{iK} e^{rT_C} - \sum_s \sum_t \omega_{ist} + \gamma \sum_s \sum_t \beta_{ist} \geq 0\right\} \perp Q^O_i \geq 0 \tag{11}$$

$$\left\{Q^O_i - Q^O_{ist} \geq 0\right\} \perp \omega_{ist} \geq 0 \quad \forall s \in \Omega \quad \forall t \in \mathcal{T} \tag{12}$$

$$\left\{\overline{Q_i} - Q^O_{ist} - Q^{Dh}_{ist} \geq 0\right\} \perp \mu_{ist} \geq 0 \quad \forall s \in \Omega \quad \forall t \in \mathcal{T} \tag{13}$$

$$\left\{\left(N_{st} - K + f_{iK} e^{rT_C} - \gamma \sum_{m \in A} Q^O_m\right) \geq 0\right\} \perp \beta_{ist} \geq 0 \quad \forall s \in \Omega \quad \forall t \in \mathcal{T} \tag{14}$$

$$\left\{T Q^O_i e^{rT_C} - e^{rT_C} \sum_s \sum_t \beta_{ist} \geq 0\right\} \perp f_{iK} \geq 0. \tag{15}$$

Every producer k of set B participates only in the day-ahead market.

The optimization problem and related KKT conditions of each producer k of set B are given in the Appendix.

By substituting λ_{st} from (6) into the objective function (3), the optimization problem (3)–(8) can be rewritten as a quadratic programming. The Hessian matrix of this quadratic programming is positive semidefinite on the nonnegative orthant. Consequently, the optimization problem of each producer is convex on its feasible set. Therefore, KKT conditions are sufficient conditions for optimality. The equilibrium of the joint option and day-ahead markets can be calculated by solving the set of KKT conditions of optimization problems of all producers.

C. Transmission-Constrained Model

In this section, transmission constraints are considered in the proposed model as the presented model in [25]. Hobbs [25] first presents a bilateral model for electricity market. This model considers an arbitrary hub node. It assumes that all generation power passes through the hub node. Transmission system operator (TSO) charges producers a congestion-based wheeling fee W_j \$/MWh for transmitting power from the hub node to node j. In the presented bilateral model in [25], nodal prices may be not equal even there is no congestion. To convert this bilateral model to a POOLCO model, it is assumed that there are some arbitragers that buy electricity from busses with low nodal prices and sell it to busses with high nodal prices [25]. The optimization problem of producer i in set A considering transmission constraints is formulated as follows:

$$\max_{Q^O_{ist}, Q^{Dh}_{ist}, Q^O_i, f_{iK}, \lambda^{hub}_{st}, y_{st,j}} E(\pi_i) = -T f_{iK} e^{rT_C} Q^O_i + \sum_{s=1}^{S} \sum_{t=t_0}^{t_T} p_s \left(Q^O_{ist}K + Q^{Dh}_{ist}\ \lambda_{st,J_i} - \left(a_i(Q^O_{ist} + Q^{Dh}_{ist}) + \frac{1}{2} b_i (Q^O_{ist} + Q^{Dh}_{ist})^2\right)\right) \tag{16}$$

s.t. :

$$Q^O_{ist} + Q^{Dh}_{ist} \leq \overline{Q_i} \;\; \forall s \in \Omega \quad \forall t \in \mathcal{T} : \mu_{ist} \tag{17}$$

$$Q^O_{ist} \leq Q^O_i \;\; \forall s \in \Omega \quad \forall t \in \mathcal{T} : \omega_{ist} \tag{18}$$

$$\lambda_{st,J_i} = N_{st,J_i} - \gamma_{J_i}\left(\sum_{m \in A | m @ J_i}(Q^O_{mst} + Q^{Dh}_{mst}) + \sum_{v \in B | v @ J_i} Q^{Dh}_{vst} + y_{st,J_i}\right) \forall s \in \Omega\, \forall t \in \mathcal{T} : \theta_{st,J_i} \tag{19}$$

$$K - f_{iK} e^{rT_C} \leq \mathcal{G}_{st}\big(\sum_{m\in A} Q^O_m\big) \ \forall s\in\Omega \ \forall t\in\mathcal{T} : \beta_{ist} \tag{20}$$

$$\lambda_{st,j} = \lambda^{hub}_{st} + W_{st,j} \ \forall s\in\Omega \quad \forall t\in\mathcal{T} \quad \forall j\in\mathcal{N} : \alpha_{ist,j} \tag{21}$$

$$\sum_{j\in\mathcal{N}} y_{st,j} = 0 \quad \forall s\in\Omega \quad \forall t\in\mathcal{T} : \xi_{ist} \tag{22}$$

$$Q^O_{ist} \geq 0, Q^{Dh}_{ist} \geq 0, Q^O_i \geq 0, f_{iK} \geq 0 \quad \forall s\in\Omega \quad \forall t\in\mathcal{T} \tag{23}$$

where $\mathcal{N}$ is the set of nodes of the network, J_i is the node that producer i is connected to it, $\lambda_{st,j}$ is the day-ahead price of node j at hour t of scenario s, λ^{hub}_{st} is a day-ahead price at hub node at hour t of scenario s, $W_{st,j}$ is the wheeling fee for transmitting power from hub node to node j at hour t of scenario s, $y_{st,j}$ is the net amount of power sold by arbitragers to node j at hour t of scenario s and is equal to the power that is transmitted to bus j through transmission lines at hour t of scenario s [25], $N_{st,j}$ and γ_j are the coefficients of inverse demand function at node j at hour t of scenario s, $\mathcal{G}_{st}(.)$ is the aggregated inverse demand function of all loads of network at hour t of scenario s, $\{m \in A|m@j\}$ is the subset of producers in set A that are connected to bus j, and $\alpha_{ist,j}$ and ξ_{ist} are the dual variables of constraints (21) and (22).

The KKT conditions of each producer i in set A considering transmission constraints are as follows:

$$\begin{aligned}&\big\{p_s\big(-K + \gamma_{J_i} Q^{Dh}_{ist} + a_i + b_i(Q^O_{ist} + Q^{Dh}_{ist})\big) + \omega_{ist} + \mu_{ist}\\ &\quad - \gamma_{J_i}\alpha_{ist,J_i} \geq 0\big\} \perp Q^O_{ist} \geq 0 \ \forall s\in\Omega \quad \forall t\in\mathcal{T}\end{aligned} \tag{24}$$

$$\begin{aligned}&\Bigg\{p_s\Bigg(-N_{st,J_i} + \gamma_{J_i}\Bigg(\sum_{m\in A|m@J_i}(Q^O_{mst} + Q^{Dh}_{mst})\\ &\qquad + \sum_{v\in B|v@J_i} Q^{Dh}_{vst} + y_{st,J_i}\Bigg) + \gamma_{J_i} Q^{Dh}_{ist} + a_i\\ &\qquad + b_i(Q^O_{ist} + Q^{Dh}_{ist})\Bigg)\\ &\quad + \mu_{ist} - \gamma_{J_i}\alpha_{ist,J_i} \geq 0\Bigg\} \perp Q^{Dh}_{ist} \geq 0 \quad \forall s\in\Omega \ \forall t\in\mathcal{T}\end{aligned} \tag{25}$$

$$\begin{aligned}&\Bigg\{T f_{iK} e^{rT_C} - \sum_s\sum_t\Bigg(\beta_{ist}\frac{\partial\mathcal{G}_{st}\big(\sum_{m\in A} Q^O_m\big)}{\partial Q^O_i}\Bigg)\\ &\quad - \sum_s\sum_t \omega_{ist} \geq 0\Bigg\} \perp Q^O_i \geq 0\end{aligned} \tag{26}$$

$$p(s)\gamma_{J_i} Q^{Dh}_{ist} - \gamma_{J_i}\alpha_{ist,J_i} + \xi_{ist} = 0 \ \forall s\in\Omega \quad \forall t\in\mathcal{T} \tag{27}$$

$$\begin{aligned}&-\gamma_j\alpha_{ist,j} + \xi_{ist} = 0 \ \forall s\in\Omega \quad \forall t\in\mathcal{T}\\ &\quad \forall j\in\mathcal{N} \,\&\, j\neq J_i\end{aligned} \tag{28}$$

$$\sum_{j\in\mathcal{N}} y_{st,j} = 0 \quad \forall s\in\Omega \quad \forall t\in\mathcal{T} \tag{29}$$

$$\sum_{j\in\mathcal{N}} \alpha_{ist,j} = 0 \quad \forall s\in\Omega \quad \forall t\in\mathcal{T} \tag{30}$$

$$\lambda_{st,j} = \lambda^{hub}_{st} + W_{st,j} \ \ \forall s\in\Omega \quad \forall t\in\mathcal{T} \quad \forall j\in\mathcal{N} \tag{31}$$

$$\begin{aligned}&\Big\{-K + f_{iK} e^{rT_C} + \mathcal{G}_{st}\Bigg(\sum_{m\in A} Q^O_m\Bigg) \geq 0\Big\} \perp \beta_{ist} \geq 0\\ &\forall s\in\Omega \quad \forall t\in\mathcal{T}\end{aligned} \tag{32}$$

and KKT conditions (12), (13), and (15).

The optimization problem of producer k in set B and its KKT conditions are given in the Appendix.

TSO maximizes its profit considering transmission constraints [25]. The optimization problem of TSO is formulated as follows [25]:

$$\max_{\boldsymbol{Y_{st}},\boldsymbol{\delta_{st}}} \boldsymbol{W_{st}}^T \boldsymbol{Y_{st}} \tag{33}$$

s.t. :

$$V_r^2 \boldsymbol{B}\boldsymbol{\delta_{st}} = -\boldsymbol{Y_{st}} \quad \forall s\in\Omega \quad \forall t\in\mathcal{T} : \boldsymbol{\psi_{st}} \tag{34}$$

$$V_r^2 \boldsymbol{H}\boldsymbol{\delta_{st}} \leq \boldsymbol{T}^+ \ \forall s\in\Omega \quad \forall t\in\mathcal{T} : \boldsymbol{\zeta^+_{st}} \tag{35}$$

$$V_r^2 \boldsymbol{H}\boldsymbol{\delta_{st}} \geq \boldsymbol{T}^- \ \forall s\in\Omega \quad \forall t\in\mathcal{T} : \boldsymbol{\zeta^-_{st}} \tag{36}$$

$$\boldsymbol{\delta_{st}} \leq \boldsymbol{\delta}^+ \ \forall s\in\Omega \quad \forall t\in\mathcal{T} : \boldsymbol{\nu^+_{st}} \tag{37}$$

$$\boldsymbol{\delta_{st}} \geq \boldsymbol{\delta}^- \ \forall s\in\Omega \quad \forall t\in\mathcal{T} : \boldsymbol{\nu^-_{st}} \tag{38}$$

where V_r is rated voltage of the system, $\boldsymbol{\delta_{st}}$ is bus phase angle vector at hour t of scenario s, $\boldsymbol{Y_{st}}$ is vector of $y_{st,j}$ for $j\in\mathcal{N}$, $\boldsymbol{W_{st}}$ is vector of $W_{st,j}$ for $j\in\mathcal{N}$, $\boldsymbol{B}$ and $\boldsymbol{H}$ are dc power flow matrices, $\boldsymbol{T}^+$ and $\boldsymbol{T}^-$ are vectors of upper and lower bounds of transmission lines, $\boldsymbol{\delta}^+$ and $\boldsymbol{\delta}^-$ are upper and lower bounds of bus phase angle vector, and $\boldsymbol{\psi_{st}}$, $\boldsymbol{\zeta^+_{st}}$, $\boldsymbol{\zeta^-_{st}}$, $\boldsymbol{\nu^+_{st}}$, and $\boldsymbol{\nu^-_{st}}$, are vectors of dual variables of constraints (34) to (38), respectively. Constraints (34) indicate power flow equations, constraints (35) and (36) indicate lines power flow constraints, and constraints (37) and (38) indicate voltages phase angle constraints.

The KKT conditions of TSO are as follows:

$$-\boldsymbol{W}_{st} + \boldsymbol{\psi}_{st} = 0 \ \forall s\in\Omega \quad \forall t\in\mathcal{T} \tag{39}$$

$$\begin{aligned}&V_r^2 \boldsymbol{B}^T\boldsymbol{\psi_{st}} + V_r^2 \boldsymbol{H}^T\boldsymbol{\zeta^+_{st}} - V_r^2 \boldsymbol{H}^T\boldsymbol{\zeta^-_{st}} + \boldsymbol{\nu^+_{st}} - \boldsymbol{\nu^-_{st}} = 0\\ &\forall s\in\Omega \quad \forall t\in\mathcal{T} \quad \forall j\in\mathcal{N}\end{aligned} \tag{40}$$

$$V_r^2 \boldsymbol{B}\boldsymbol{\delta_{st}} = -\boldsymbol{Y_{st}} \quad \forall s\in\Omega \quad \forall t\in\mathcal{T} \tag{41}$$

$$\big\{\boldsymbol{T}^+ - V_r^2 \boldsymbol{H}\boldsymbol{\delta_{st}} \geq 0\big\} \perp \boldsymbol{\zeta^+_{st}} \geq 0 \ \forall s\in\Omega \quad \forall t\in\mathcal{T} \tag{42}$$

$$\big\{V_r^2 \boldsymbol{H}\boldsymbol{\delta_{st}} - \boldsymbol{T}^- \geq 0\big\} \perp \boldsymbol{\zeta^-_{st}} \geq 0 \ \forall s\in\Omega \quad \forall t\in\mathcal{T} \tag{43}$$

$$\big\{\boldsymbol{\delta}^+ - \boldsymbol{\delta_{st}} \geq 0\big\} \perp \boldsymbol{\nu^+_{st}} \geq 0 \ \forall s\in\Omega \quad \forall t\in\mathcal{T} \tag{44}$$

$$\big\{\boldsymbol{\delta_{st}} - \boldsymbol{\delta}^- \geq 0\big\} \perp \boldsymbol{\nu^-_{st}} \geq 0 \ \forall s\in\Omega \quad \forall t\in\mathcal{T}. \tag{45}$$

Equilibrium of the joint option and day-ahead markets can be calculated by solving the set of KKT conditions of optimization problems of all producers and TSO.

D. *Option Contract Area*

Financial markets operators usually exercise restrictions to premium prices [32]. Market regulators are willing to study the behavior of market at different premium prices before applying any restrictions. During a specified day of trading period, premium prices change in small range. To study the behavior of market at different premium prices, it is assumed that premium prices of all producers are equal at each day of trading period and it is an exogenous variable as strike price. In this paper, the set of strike price and premium price pairs at which option contracts are concluded is referred to as *option contract area*. It is desire for market regulator to determine the option contract area. To this end, market equilibrium is computed for different pairs of strike–premium prices and the pairs of strike–premium prices at which option contracts are concluded are determined. To compute market equilibrium, f is considered as an exogenous variable in proposed models presented in Sections II-B and II-C. The following lemma specifies the option contract area in strike–premium prices plane, if transmission constraints are ignored.

Lemma 1: The option contract area can be defined by the following inequalities on the strike price and premium price plane:

$$\left\{(f,K)|Te^{rT_C}f/\eta+\overline{\lambda^0}<K<N^{\min}+fe^{rT_C}\right\} \tag{46}$$

where $N^{\min}$ is the minimum of N_{st} over different hours and scenarios of the delivery period, f is the premium price in the option market, and η and $\overline{\lambda^0}$ are computed as follows:

$$\eta=\sum_{\{s,t|K>\lambda^0_{st}\}}p_s,\overline{\lambda^0}=(1/\eta)\sum_{\{s,t|K>\lambda^0_{st}\}}p_s\lambda^0_{st} \tag{47}$$

where λ^0_{st} is the electricity price of day-ahead market at hour t of scenario s if no option contract is concluded.

Proof: Consider a day-ahead electricity market with an option electricity market. Suppose a standard option has been defined for a specified delivery period, and market regulator has considered a single strike price for this option. Consider a fixed premium price f, and assume market regulator is willing to study the behavior of producers and consumers at this fixed f by changing the strike price. Suppose the strike price is low and no producer is willing to buy option. Now, assume market regulator increases the strike price with small step size until at least one option contract is concluded by a producer in the option market. Suppose producer i decides to sell a part of its power in the option market and consequently buys Q_i^O mega watt option at strike price K and premium price f. Assume producer i exercises Q^O_{ist} at hour t of scenario s in the delivery period. The expected surplus profit that producer i gains if buys Q_i^O option at strike price K and premium price f is shown with $E(S\pi_i)$ and can be calculated as follows:

$$E(S\pi_i)=-Q_i^OTfe^{rT_C}+\sum_{s=1}^{S}\sum_{t=t_0}^{t_T}p_sQ^O_{ist}(K-\lambda^0_{st}). \tag{48}$$

The first term of (48) indicates the total premium that producer i must pay for buying option. The second term of (48) is the expected revenue of producer i if he or she exercises Q^O_{ist} at hour t of scenario s in the option market. The last term of (48) is negative of the expected revenue of producer i if he or she sells Q^O_{ist} at hour t of scenario s to the day-ahead market instead of the option market. Total expected surplus of all producers is equal to

$$E(S\pi)=-Q^OTfe^{rT_C}+\sum_{s=1}^{S}\sum_{t=t_0}^{t_T}p_sQ^O_{st}(K-\lambda^0_{st}) \tag{49}$$

where Q^O is the total volume of option contracts that is concluded at strike price K and premium price f, and Q^O_{st} is the total volume of exercised option contracts at hour t of scenario s of the delivery period. Q^O_{st} is equal to zero if $K\le\lambda^0_{st}$, and is equal to Q^O if $K>\lambda^0_{st}$. Total expected surplus of all producers can be rewritten as follows:

$$E(S\pi)=-Q^OTfe^{rT_C}+K\sum_{\{s,t|K>\lambda^0_{st}\}}p_sQ^O - \sum_{\{s,t|K>\lambda^0_{st}\}}p_sQ^O\lambda^0_{st}. \tag{50}$$

In the option contract area, total expected surplus of all producers and total concluded volume of option contracts are greater than zero. Assuming the right-hand side of (50) is greater than zero yields the lower bound of the option contract area as follows:

$$K>Te^{rT_C}f/\eta+\overline{\lambda^0} \tag{51}$$

where η is equal to sum of p_s over all hours and scenarios at which $K>\lambda^0_{st}$, and $\overline{\lambda^0}$ is the expected value of day-ahead market price over all hours and scenarios that $K>\lambda^0_{st}$. Variables η and $\overline{\lambda^0}$ are computed as (47).

On the other hand, price elasticity of demand restricts total concluded volume of option contracts. As long as total payment for buying 1 MWh electricity from option market is less than the value of 1 MWh electricity for consumers at EHESDP, i.e., as long as $(K-fe^{rT_C})<N_{st}\ \forall s\in\Omega,\forall t\in\mathcal{T}$, consumers are willing to conclude option contract in the option market. Hence, for every strike price K and premium price f that satisfy the following inequalities:

$$(K-fe^{rT_C})<N_{st}\qquad\forall s\in\Omega\quad\forall t\in\mathcal{T} \tag{52}$$

there is a $Q_i^O>0$ that satisfies (7). Conditions (52) can be abstracted as follows:

$$K<fe^{rT_C}+N^{\min}. \tag{53}$$

Option contract area is the intersection of (51) and (53), or the area that is defined by inequalities (46). ■

The borders of option contract area can be specified with $K_{\inf}=Te^{rT_C}f/\eta+\overline{\lambda^0}$ and $K_{\sup}=fe^{rT_C}+N^{\min}$ and are referred to as *low* and *high strike price borders*, respectively. Equation (46) gives market regulator a formula to compute strike prices at which producers and consumers are willing to conclude option contract. These strike prices are referred to as efficient strike prices. This formula only depends to the parameters of the associated day-ahead market before this specified option is put in the option market. Financial market operator can define efficient strike prices for a specified option based on the historical data of the associated day-ahead market in a similar delivery period. In this case, there is only one scenario, η is the number of hours that strike price is greater than day-ahead market price, $\overline{\lambda^0}$ is the average of day-ahead market price over all hours at which strike price is greater than day-ahead market price, and $N^{\min}$ is the minimum of N_{st} over different hours of the considered similar delivery period.

For each strike price, option contract area gives a range for premium price. Therefore, computing option contract area can be considered as an option pricing method.

III. Case Study

In this section, the proposed model applied to a 4-unit system. Capacities of the producers and coefficients of their marginal cost functions are given in Table I. Assume producers 1 and 2 participate in the both option and day-ahead markets and

TABLE I
CHARACTERISTICS OF ALL PRODUCERS

	Number of producer	Coefficients of marginal cost functions		Generation capacity (GW)
		a_i (\$/MWh)	b_i (\$/MW^{2}h)	
Set A	1	18.108	0.001483	12.00
	2	7.3137	0.003739	11.40
Set B	3	19.066	0.001776	8.721
	4	12.943	0.153700	0.558

TABLE II
EXPECTED VALUE AND STANDARD DEVIATION OF INTERCEPT OF INVERSE DEMAND FUNCTION AT DIFFERENT HOURS OF THE DELIVERY PERIOD

Hour t	1	2	3	4	5	6	7	8	9	10
$E(N_t)$ (\$/MWh)	44.1	39	48.9	48	48.3	46.8	48	42.9	39.9	45.9
Std(N_t) (\$/MWh)	0.09	0.07	0.15	0.14	0.14	0.11	0.13	0.09	0.07	0.11

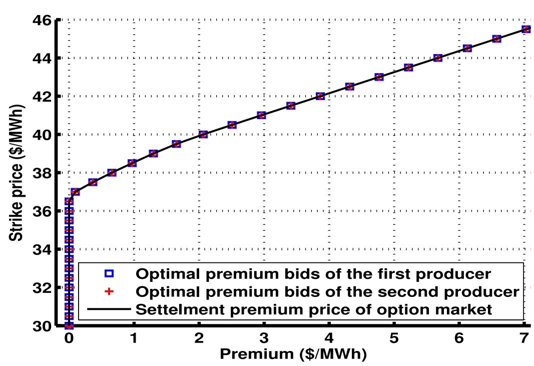

Fig. 2. Optimal premium bids of the first and second producers and the related settlement premium price.

producers 3 and 4 only participate in the day-ahead market, i.e., producers 1 and 2 are in set A and producers 3 and 4 are in set B, as shown in Table I. Suppose delivery period of the understudy option contracts consists of a single hour of ten consecutive days. Suppose expected value and standard deviation of N_t are estimated for each hour of the delivery period, and are given in Table II. Although demand changes in the delivery period, it is assumed that demand function is affine and its slope remains constant and equal to $\gamma = \$\ -0.0003/\text{MW}^2\text{h}$.

In order to consider uncertainty in demand, five scenarios are defined for demand function in the delivery period. In each scenario, N_{st} is assigned to each hour of the delivery period based on the distribution functions of demand at different hours. Assume contract period is one year or $T_C = 1$.

A. No-Transmission Model

In order to study the impacts of strike prices on the option and day-ahead markets, it is assumed that strike price varies from \$30/MWh to \$46/MWh with step size \$1/MWh. At each strike price, equilibrium of the joint option and day-ahead markets is computed assuming premium prices are endogenous variables. Optimal premium prices of the first and second producers at the market equilibrium are shown in Fig. 2. As shown in this figure, the optimal premium bids of the first and second producers

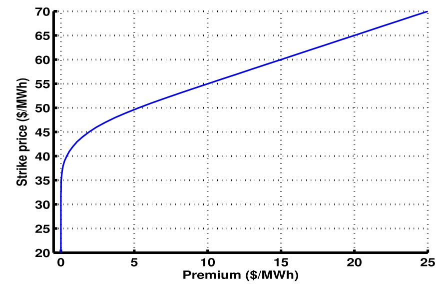

Fig. 3. Put option pricing for *NSW's Base Load Strip Options, Calendar Year 2017,* in Australia Securities Exchange at Febraury 28, 2016.

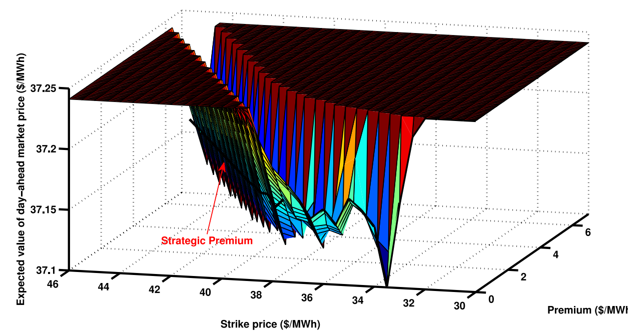

Fig. 4. Expected value of day-ahead market price over delivery period.

are equal at the equilibrium of the joint option and day-ahead markets.

In some option markets, a settlement premium price is computed for each day of trading period for mark-to-marketing process, i.e., for valuating options by the most recent market prices [32]. Settlement premium price of a day is equal to the weighted average of premium price of the option contracts that are traded on that day or on part of that day [32]. Since premium prices of all producers are equal at market equilibrium, settlement premium price is equal to premium price of each producer at market equilibrium, as shown in Fig. 2. Hence, computing the equilibrium of the joint option and day-ahead markets can be considered as a method for option pricing. The illustrated curve in Fig. 2 shows option prices for different strike prices in this case study. The result of put option pricing in Australian Securities Exchange (ASX) at February 28, 2016 is illustrated in Fig. 3 [32]. Fig. 3 shows the put option pricing for NSWs Base Load Strip Options, Calendar Year 2017. Comparison of Figs. 2 and 3 shows that the strike price and premium price curve that is obtained from the proposed method is very similar to the actual one that is obtained from the historical data of ASX.

In order to determine option contract area, strike price varies from \$30/MWh to \$46/MWh with step size \$0.5/MWh, and at each strike price, premium price varies from \$0/MWh to \$7/MWh with step size \$0.1/MWh. Then, for each pair of premium and strike prices, equilibrium point of the joint option and day-ahead markets is calculated assuming premium price is an exogenous variable.

Expected values of day-ahead market price over all hours and scenarios for different premium and strike prices are illustrated in Fig. 4. Now suppose premium price is a strategic endogenous variable and it is determined so that the profits of producers are maximized. In this case, the expected value of day-ahead market price is as the solid line shown in Fig. 4.

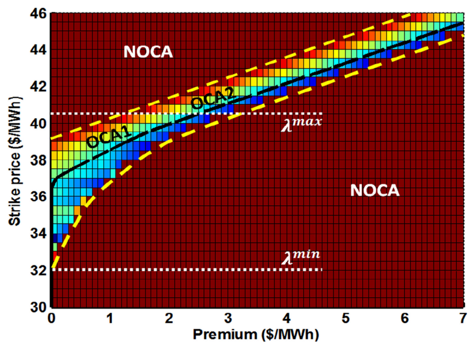

Fig. 5. Projection of Fig. 3 on the premium price and strike price plane.

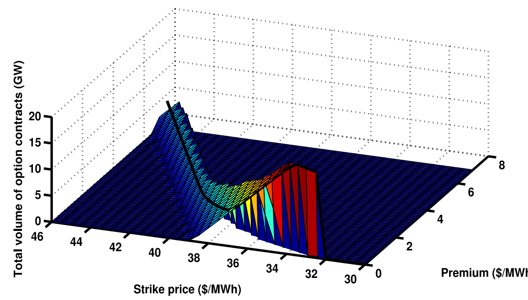

Fig. 6. Total volume of concluded option contracts.

Two segments of a horizontal plane are observed in Fig. 4. According to the simulation results, no option contract is concluded in these horizontal segments. In these segments, the expected value of the day-ahead market price is constant and is equal to the expected value of the day-ahead market price when no option is traded. The projection of Fig. 4 on the premium price and strike price plane is demonstrated in Fig. 5. Since no option contract is concluded in the horizontal segments, the projection of these segments on the premium price and strike price plane is referred to as *no option contract area* and it is specified with NOCA in Fig. 5. Low and high strike price borders are specified with two dashed curves in Fig. 5. Simulation results show that total concluded volume of the option contracts that their strike prices are between $K_{\inf}$ and $K_{\sup}$ is greater than zero at the equilibrium of the joint option and day-ahead markets. Hence, the area between $K_{\inf}$ and $K_{\sup}$ on the premium price and strike price plane is referred to as *option contract area* and is specified with OCA. In Fig. 5, $\lambda^{\max}$ and $\lambda^{\min}$ are the maximum and minimum of λ_{st} over all hours and scenarios. According to (46), the high strike price border is an affine function with slope e^{rT_C}. In this case study, for interest rate of 10% and contract period of one year, the slope of the high strike price border is approximately 1.1. The high strike price border that is drawn based on the simulation results in Fig. 5 verifies the presented formula for the high strike price border, i.e., $K_{\sup} = fe^{rT_C} + N^{\min}$. Consider the low strike price border and assume K increases on this border. As K increases, $K - \lambda_{st}$ increases, the chance of exercising option contracts increases, and consequently η converges to T. As K exceeds $\lambda^{\max}$, all option contracts are exercised, η gets equal to T, and based on (46) the low strike price border becomes an affine function parallel to the high strike price border. The low strike price border that is drawn based on the simulation results in Fig. 5 verifies the presented formula for the low strike price border, i.e., $K_{\inf} = Te^{rT_C} f/\eta + \overline{\lambda^0}$. As it is illustrated in Fig. 5, the option contract area is partitioned into OCA1 and OCA2. In OCA2, strike prices are greater than $\lambda^{\max}$, and consequently option contracts are exercised at all hours of all scenarios of the delivery period. In OCA1 strike prices are between $\lambda^{\min}$ and $\lambda^{\max}$, and consequently option contracts are exercised at some hours of some scenarios of the delivery period.

Consider a premium price and a small strike price, and increase the strike price. As strike price slightly exceeds the strike price of the low strike price border and enters into OCA, based on (51) the strike price exceeds the sum of day-ahead market price and future value of premium price at some hours of some scenarios, or $K > fe^{rT_C} + \lambda_{st}$ at some hours of some scenarios. Hence, it is profitable for producers to exercise option contracts at these hours of these scenarios. Therefore, producers conclude option contracts in the option market in advance of day-ahead market. After concluding option contracts, every producer i attends in the day-ahead market with total capacity of $\overline{Q}_i - Q_i^O$ to supply the residual demand. This leads to decrease of day-ahead market price and consequently increase of total consumption and generation. Therefore, expected value of day-ahead market price over all hours and scenarios falls if strike price slightly exceeds the strike price of the low strike price border and enters into OCA.

The maximum volume of concluded option contracts are restricted by demand constraint. According to (7), consumers do not consume more than a specified amount of power in a specified premium and strike prices. Consider a specified premium price and a strike price between the low and high strike price borders, and increase the strike price. Based on (7), consumers decrease the volume of their option contracts as strike price increases and consequently the expected value of day-ahead market price increases, as shown in Fig. 4. As strike price reaches to high strike price border, no option contract is concluded by consumers in the option market.

Total volume of concluded option contracts are shown in Fig. 6. Total volume of concluded option contracts for the case that the premium price is a strategic endogenous variable is shown by a solid line in this figure. Areas OCA1, OCA2, and NOCA are identifiable in this figure. Since in OCA2, strike price is greater than the maximum of the day-ahead market price, consumers do not conclude option contract in OCA2 as much as in OCA1, as shown in Fig. 6. Note that although the strike price is greater than the maximum of the day-ahead market price in OCA2, premium price is also high in this area and encourages consumers to conclude option contract in this area.

If the values of strike prices are not determined properly or too much strike prices for a specified delivery period are determined, competition focus on each strike price decreases. Hence, the values of strike prices should be in a specified range and the number of strike prices should be limited. Based on Fig. 6, by defining a limited number of strike prices in the OCA1 and putting them in the option market, competition focus on these strike prices and consequently traded volume of option contracts increase.

Expected value of total social welfare at the equilibrium of the joint option and day-ahead markets for different premium and strike prices is illustrated in Fig. 7. Expected value of total social welfare for the case that the premium price is a strategic endogenous variable is shown by a solid line in this figure. Areas OCA1, OCA2, and NOCA are identifiable in Fig. 7. Since expected value of total consumption increases in the option contract area, expected value of total social welfare increases in this area, as shown in this figure. Fig. 7 shows that the maximum expected value of total social welfare is achieved at the strike

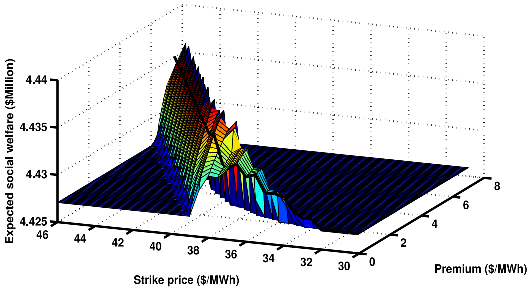

Fig. 7. Expected value of total social welfare in the both option and day-ahead markets.

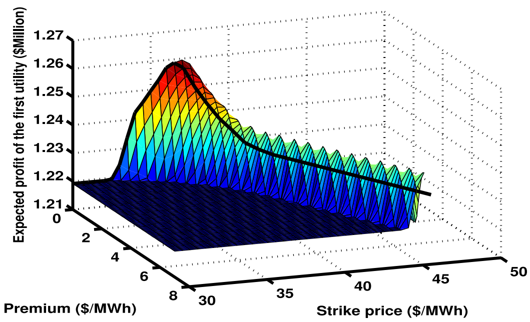

Fig. 8. Expected profit of the first producer in the both option and day-ahead markets.

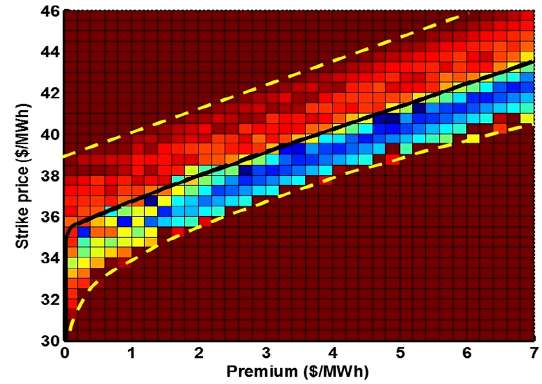

Fig. 9. Option contract area of congested case.

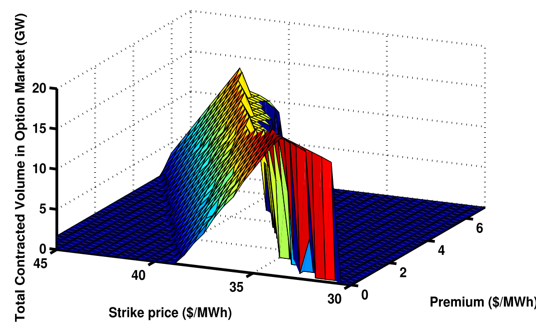

Fig. 10. Total volume of concluded option contracts in congested case.

prices that are slightly greater than the strike price of the low strike price border in OCA2. From Fig. 7, it is also found that the expected value of total social welfare in OCA1 is not as high as the maximum expected value of total social welfare in OCA2.

Expected profit of the first producer for different premium and strike prices is shown in Fig. 8. Expected profit of the first producer for the case that the premium price is a strategic endogenous variable is shown by a solid line in this figure. Areas OCA1, OCA2, and NOCA are also identifiable in this figure. According to Fig. 8, the expected profit of the first producer increases in the option contract area. Fig. 8 also shows that the profit of the first producer in OCA1 is greater than its profit in OCA2. Simulation results show that although the expected profit of the first and the second producers that participate in the both option and day-ahead markets increase in the option contract area, the expected profit of the third and fourth producers that only participate in the day-ahead market decrease in the option contract area. This means strategic participating of the first and the second producers in the option market leads to decrease in the expected profit of the producers that do not participate in the option market. This is an encouraging signal for producers to participate in the both markets strategically.

B. Transmission-Constrained Model

In this section, a three area power system is considered in order to study the impacts of option market on the operation of joint option and day-ahead markets considering transmission constraints. The first and fourth producers are in area 1, the second producer is in area 2, and the third producer is in area 3. Each pair of areas is interconnected by a set of parallel tie-lines. Impedance of each set of parallel tie-lines is equal to 20 Ω at rated voltage 500 KV. Each area has a load with linear demand function. Intercepts of inverse demand function of three areas are equal. Expected values and standard deviations of intercepts of inverse demand functions of each area for different hours of delivery period are given in Table II. Slopes of inverse demand functions of areas 1–3 are \$–0.002/MW^{2}h, \$$-0.002$/MW^{2}h, and \$ -0.00043/MW^{2}h, respectively. These slopes are considered such that the aggregated inverse demand function of the system is equal to the inverse demand function of the previous subsection.

First, it is assumed that each set of tie-lines has unlimited capacity and simulations of Section III-A are repeated. Simulation results for unlimited tie-lines case are exactly the same as the results of Section III-A. Then, it is assumed that total capacity of parallel tie-lines between areas 1 and 3 is limited to 3 GW and the simulations are repeated. In this case, parallel tie-lines between areas 1 and 3 are congested in some scenarios of some hours of delivery period. OCA for congested case is illustrated in Fig. 9. According to (53), high strike price border does not depend on day-ahead price and consequently remains constant in congested case in comparison to uncongested case, as shown in Figs. 9 and 5. Due to having more than one marginal generator in congested case, day-ahead price of some busses and consequently $\overline{\lambda^0}$ decrease in congested case in comparison to uncongested case. Hence, according to (51), K decreases for each premium price. This leads to expansion of OCA from the low strike price border side, as shown in Figs. 9 and 5.

Since congestion prevents from dispatching of some producers in day-ahead market, the producers are encouraged to hedge themselves by buying option contracts. This leads to increase in demand for concluding option contracts and consequently increase in concluded option volume and premium price at each strike price. Hence, as shown in Fig. 10, total volume of concluded option contracts increases in congested case in comparison to uncongested case.

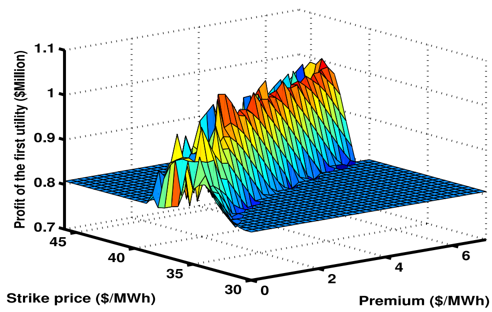

Fig. 11. Expected profit of the first producer in the both option and day-ahead markets in congested case.

Expected profit of the first producer in the both option and day-ahead markets in congested case is shown in Fig. 11.

By congesting transmission lines 1–3, expected profit of the first producer decreases from 2.2 $million to about 0.78$million in NOCA. However, the expected profit of the first producer in both option and day-ahead markets increases in OCA by concluding and exercising option contracts, as shown in Fig. 11.

IV. Conclusion

In this paper, the impacts of strike and premium prices of put option contracts on the option and day-ahead markets are studied. Simulation results show that strategic participation of some producers in the both option and day-ahead markets leads to decrease in expected profits of strategic producers that only participate in the day-ahead market. Using the presented model, financial market operators can define a proper range for strike prices so that a high volume of option is traded. Congestion in transmission lines leads to expansion of OCA and increase in total volume of concluded option contracts. Usually in option markets, an estimation of option price is announced for market participants. Computing settlement premium price using the presented equilibrium model with strategic premium bids and determining the option contract area using the presented equilibrium model with nonstrategic premium bids can be considered as option pricing method, which gives an estimation of premium price and a range for premium price, respectively.

Appendix
Formulation of Producers in Set B

A. No-Transmission Model

Optimization problem of producer k in set B ignoring transmission constraints is formulated as follows:

$$\max_{Q_{kst}^{Dh}} E(\pi_k) = \sum_{s=1}^{S}\sum_{t=t_0}^{t_T} p_s \left(Q_{kst}^{Dh}\lambda_{st} - \left(a_k Q_{kst}^{Dh} + \frac{1}{2} b_k {Q_{kst}^{Dh}}^2\right)\right) \tag{54}$$

s.t. :

$$Q_{kst}^{Dh} \le \overline{Q_k} \quad \forall s \in \Omega \quad \forall t \in \mathcal{T} : \mu_{kst} \tag{55}$$

$$\lambda_{st} = N_{st} - \gamma\left(\sum_{m\in A}(Q_{mst}^{O} + Q_{mst}^{Dh}) + \sum_{v\in B} Q_{vst}^{Dh}\right)$$
$$\forall s \in \Omega \quad \forall t \in \mathcal{T} : \theta_{st} \tag{56}$$

$$Q_{kst}^{Dh} \ge 0 \, \forall s \in \Omega \quad \forall t \in \mathcal{T}. \tag{57}$$

The KKT optimality conditions of each producer k of set B are as follows:

$$\Bigg\{p_s\Bigg(-N_{st} + \gamma\left(\sum_{m\in A}(Q_{mst}^{O} + Q_{mst}^{Dh}) + \sum_{v\in B} Q_{vst}^{Dh}\right) + \gamma Q_{kst}^{Dh} + a_k + b_k Q_{kst}^{Dh}\Bigg) + \mu_{kst} \ge 0\Bigg\} \perp Q_{kst}^{Dh} \ge 0$$
$$\forall s \in \Omega \quad \forall t \in \mathcal{T} \tag{58}$$

$$\{\overline{Q_k} - Q_{kst}^{Dh} \ge 0\} \perp \mu_{kst} \ge 0 \quad \forall s \in \Omega \quad \forall t \in \mathcal{T}. \tag{59}$$

B. Transmission-Constrained Model

Optimization problem of producer k in set B considering transmission constraints is formulated as follows:

$$\max_{Q_{kst}^{Dh}, \lambda_{st}^{hub}, y_{st,j}} E(\pi_k)$$
$$= \sum_{s=1}^{S}\sum_{t=t_0}^{t_T} p_s \left(Q_{kst}^{Dh}\lambda_{st,J_k} - \left(a_k Q_{kst}^{Dh} + \frac{1}{2} b_k {Q_{kst}^{Dh}}^2\right)\right) \tag{60}$$

s.t. :

$$Q_{kst}^{Dh} \le \overline{Q_k} \;\; \forall s \in \Omega \quad \forall t \in \mathcal{T} : \mu_{kst} \tag{61}$$

$$\lambda_{st,J_k} = N_{st,J_k} - \gamma_{J_k}\Bigg(\sum_{m\in A|m@J_k}(Q_{mst}^{O} + Q_{mst}^{Dh}) + y_{st,J_k} + \sum_{v\in B|v@J_k} Q_{vst}^{Dh}\Bigg) \;\; \forall s \in \Omega \quad \forall t \in \mathcal{T} : \theta_{st,J_k} \tag{62}$$

$$\lambda_{st,j} = \lambda_{st}^{hub} + W_{st,j} \;\; \forall s \in \Omega, \forall t \in \mathcal{T} \quad \forall j \in \mathcal{N} : \alpha_{kst,j} \tag{63}$$

$$\sum_{j\in\mathcal{N}} y_{st,j} = 0 \quad \forall s \in \Omega \quad \forall t \in \mathcal{T} \quad : \xi_{kst} \tag{64}$$

$$Q_{kst}^{Dh} \ge 0 \quad \forall s \in \Omega \quad \forall t \in \mathcal{T}. \tag{65}$$

The KKT conditions of each producer k in set B considering transmission constraints are as follows:

$$\Bigg\{p_s\Bigg(-N_{st,J_k} + \gamma_{J_k}\Bigg(\sum_{m\in A|m@J_k}(Q_{mst}^{O} + Q_{mst}^{Dh}) + \sum_{v\in B|v@J_k} Q_{vst}^{Dh} + y_{st,J_k}\Bigg) + \gamma_{J_k} Q_{kst}^{Dh} + a_k + b_k Q_{kst}^{Dh}\Bigg) + \mu_{kst} - \gamma_{J_k}\alpha_{kst,J_k} \ge 0\Bigg\} \perp Q_{kst}^{Dh} \ge 0 \quad \forall s \in \Omega \quad \forall t \in \mathcal{T} \tag{66}$$

$$\gamma_{J_k} Q_{kst}^{Dh} - \gamma_{J_k}\alpha_{kst,J_k} + \xi_{kst} = 0 \; \forall s \in \Omega \quad \forall t \in \mathcal{T} \tag{67}$$

$$-\gamma_j \alpha_{kst,j} + \xi_{kst} = 0 \; \forall s \in \Omega \quad \forall t \in \mathcal{T} \quad \forall j \in \mathcal{N} | j \ne J_k \tag{68}$$

and (29), (30), (31), and (55).

References

[1] S. J. Deng and S. S. Oren, “Electricity derivatives and risk management,” *Energy*, vol. 31, no. 6, pp. 940–953, 2006.

[2] J. Usaola and M. A. Moreno, “Optimal bidding of wind energy in intraday markets,” in *Proc. 6th Int. Conf. Eur. Energy Market*, May 2009, pp. 1–7.

[3] J. Garca-Gonzlez *et al.*, “Connecting the intraday energy and reserve markets by an optimal redispatch,” *IEEE Trans. Power Syst.*, vol. 22, no. 4, pp. 2220–2231, Nov. 2007.

[4] “Nordpool financial market.” [Online]. Available: http://www.nordpoolspot.com/How-does-it-work/Financial-market/. Accessed on: May 30, 2016.

[5] W. Ruiqing, L. Yuzeng, and Z. Shaohua, “Analysis of supply function equilibrium in electricity markets with financial options contracts,” in *Proc. 3rd Int. Conf. Elect. Deregulation Restruct. Power Technol.*, Nanjing, China, 2008, pp. 247–251.

[6] S. Pineda and A. J. Conejo, “Using electricity options to hedge against financial risks of power producers,” *J. Modern Power Syst. Clean Energy*, vol. 1, no. 2, pp. 101–109, Sep. 2013.

[7] J. C. Hull, *Options, Futures, and Other Derivatives, Business and Economics Power System Oscillations*. Boston, MA, USA: Prentice-Hall, 2012.

[8] T. W. Gedra, “Optional forward contracts for electric power markets,” *IEEE Trans. Power Syst.*, vol. 9, no. 4, pp. 1766–1773, Nov. 1994.

[9] Y. Oum, S. S. Oren, and S. J. Deng, “Hedging quantity risks with standard power options in a competitive wholesale electricity market,” *Naval Res. Logist.*, vol. 53, no. 7, pp. 697–712, Oct. 2006.

[10] G. A. V. Snchez, J. M. Alzate, A. I. Cadena, and J. M. Benavides, “Setting up standard power options to hedge price-quantity risk in a competitive electricity market: The colombian case,” *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1493–1500, Aug. 2011.

[11] S. Pineda and A. J. Conejo, “Managing the financial risks of electricity producers using options,” *Energy Econ.*, vol. 34, no. 6, pp. 2216–2227, Nov. 2012.

[12] B. Bezerra, L. A. Barroso, and M. V. Pereira, “Bidding strategies with fuel supply uncertainty in auctions of long-term energy call options,” *IEEE Trans. Power Syst.*, vol. 26, no. 2, pp. 653–660, May 2011.

[13] P. Mathuria and R. Bhakar, “Integrated risk management model for portfolio selection in multiple markets,” in *Proc. 2014 IEEE PES Gen. Meeting*, Jul. 2014, pp. 1–5.

[14] T. S. Chung, S. H. Zhang, C. W. Yu, and K. P. Wong, “Electricity market risk management using forward contracts with bilateral options,” *Proc. Inst. Elect. Eng.—Gener. Transmiss. Distrib.*, vol. 150, no. 5, pp. 588–594, Sep. 2003.

[15] W. Ruiqing, L. Yuzeng, and Z. Shaohua, “Equilibrium for spot and options markets with capacity constraints,” in *Proc. Chin. Control Conf.*, Jul. 2008, pp. 758–762.

[16] Z. Shaohua, F. Xinhua, and W. Xian, “Effects of option contracts on electricity markets: A Cournot equilibrium analysis,” in *Proc. Asia–Pacific Power Energy Eng. Conf.*, 2012.

[17] W. Ruiqing, L. Yuzeng, and Z. Shaohua, “Joint equilibrium analysis for options and spot gaming in electricity markets with transmission constraints,” in *Proc. 7th World Congr. Intell. Control Autom.*, Jun. 2008, pp. 6497–6502.

[18] H. Bessembinder and M. L. Lemmon, “Equilibrium pricing and optimal hedging in electricity forward markets,” *J. Finance*, vol. 57, no. 3, pp. 1347–1382, 2002.

[19] H. Raouf-Sheybani and M. Oloomi-Buygi, “How does pricing of day-ahead electricity market affect put option pricing?” *Iranian J. Elect. Electron. Eng.*, vol. 12, no. 3, pp. 230–239, Sep. 2016.

[20] H. Raouf-Sheybani and M. Oloomi-Buygi, “Impacts of premium bounds on the operation of put option and day-ahead electricity markets,” *Int. J. Eng. Trans. B, Appl.*, vol. 30, no. 11, pp. 1338–1347, Nov. 2017.

[21] Y. Chen and B. F. Hobbs, “An oligopolistic power market model with tradable NOx permits,” *IEEE Trans. Power Syst.*, vol. 20, no. 1, pp. 119–129, Feb. 2005.

[22] S. J. Kazempour, A. J. Conejo, and C. Ruiz, “Generation investment equilibria with strategic producers—Part I: Formulation,” *IEEE Trans. Power Syst.*, vol. 28, no. 3, pp. 2613–2622, Aug. 2013.

[23] S. J. Kazempour and H. Zareipour, “Equilibria in an oligopolistic market with wind power production,” *IEEE Trans. Power Syst.*, vol. 29, no. 2, pp. 686–697, Mar. 2014.

[24] M. Oloomi-Buygi, H. Zareipour, and W. D. Rosehart, “Impacts of large-scale integration of intermittent resources on electricity markets: A supply function equilibrium approach,” *IEEE Syst. J.*, vol. 6, no. 2, pp. 220–232, Jun. 2012.

[25] B. F. Hobbs, “Linear complementarity models of Nash-Cournot competition in bilateral and POOLCO power markets,” *IEEE Trans. Power Syst.*, vol. 16, no. 2, pp. 194–202, May 2001.

[26] A. Yaghooti, M. Oloomi-Buygi, and H. Zareipour, “Impacts of ramp rate limits on oligopolistic opportunities in electricity markets,” *IEEE Syst. J.*, vol. 10, no. 1, pp. 127–135, Mar. 2016.

[27] M. Banaei, M. Oloomi-Buygi, and H. Zareipour, “Impacts of strategic bidding of wind power producers on electricity markets,” *IEEE Trans. Power Syst.*, vol. 31, no. 6, pp. 4544–4553, Nov. 2016.

[28] H. Song, C. C. Liu, and J. Lawarre, “Nash equilibrium bidding strategies in a bilateral electricity market,” *IEEE Trans. Power Syst.*, vol. 17, no. 1, pp. 73–79, Feb. 2002.

[29] Y. Xu and S. H. Low, “An efficient and incentive compatible mechanism for wholesale electricity markets,” *IEEE Trans. Smart Grid*, vol. 8, no. 1, pp. 128–138, Jan. 2017.

[30] B. F. Hobbs, C. B. Metzler, and J. S. Pang, “Strategic gaming analysis for electric power systems: An MPEC approach,” *IEEE Trans. Power Syst.*, vol. 15, no. 2, pp. 638–645, May 2000.

[31] D. Pozo and J. Contreras, “Finding multiple Nash equilibria in pool-based markets: A stochastic EPEC approach,” *IEEE Trans. Power Syst.*, vol. 26, no. 3, pp. 1744–1752, Aug. 2011.

[32] “Australian electricity futures and options.” [Online]. Available: https://asxenergy.com.au/products/electricity_futures. Accessed on: Sep. 30, 2015.

[33] Q. Bian and Z. Lu, “The valuation of optional financial contract in electricity market,” in *Proc. Int. Conf. Sustain. Power Gener. Supply*, Sep. 2012, pp. 1–6.

[34] M. Davison, C. L. Anderson, B. Marcus, and K. Anderson, “Development of a hybrid model for electrical power spot prices,” *IEEE Trans. Power Syst.*, vol. 17, no. 2, pp. 257–264, May 2002.

[35] B. Willems, I. Rumiantseva, and H. Weigt, “Cournot versus supply functions: What does the data tell us?” *Energy Econ.*, vol. 31, no. 1, pp. 38–47, Jan. 2009.

[36] “European electricity exchange.” [Online]. Available: https://www.eex.com/en/products/power/power-derivatives-market. Accessed on: May 30, 2016.

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